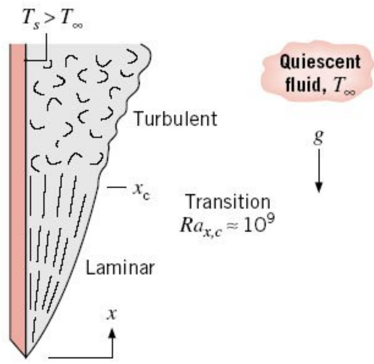
Laminar and turbulent free convection

- Like forced convection, free convection can start off as laminar and then become turbulent
- The transition is marked by a critical Rayleigh number $Ra_{x,c}$ that depends on geometry
  - e.g. $Ra_{x,c} \approx 10^9$ for vertical plates



In free convection heat transfer, the empirical correlations are generally of the form:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = CRa_L^n$$

Where the Rayleigh number,

$$Ra_L = Gr_L \Pr = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon\alpha}$$

Vertical, Inclined and Horizontal Plates:
Churchill and Chu correlation (Equation 9.26):

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Horizontal plates:

Hot Surface Up or Cold Surface Down:

$$\overline{Nu_L} = 0.54 Ra_L^{1/4} \quad 10^4 \le Ra_L \le 10^7 \quad \text{(Equations 9.30)}$$

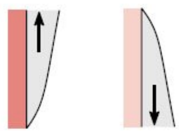
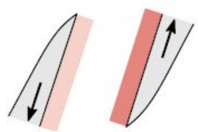
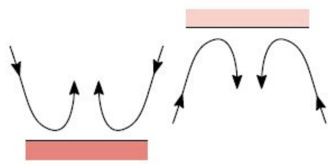
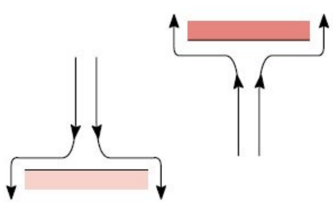
$$\overline{Nu_L} = 0.15 Ra_L^{1/3} \quad 10^7 \le Ra_L \le 10^{11} \quad \text{(Equations 9.31)}$$

Cold Surface Up or Hot Surface Down:

$$\overline{Nu_L} = 0.27 Ra_L^{1/4} \quad 10^5 \le Ra_L \le 10^{10} \quad \text{(Equations 9.32)}$$

The characteristic length for horizontal surfaces is calculated from $L_c = A_s/P$ where A_s is the surface area and P is the perimeter.

TABLE 9.2 Summary of free convection empirical correlations for immersed geometries

Geometry	Recommended Correlation	Restrictions
1. Vertical plates[a] 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \longrightarrow g\cos\theta$	$0 \leq \theta \lesssim 60°$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30	$10^4 \lesssim Ra_L \lesssim 10^7$
	Equation 9.31	$10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^5 \lesssim Ra_L \lesssim 10^{10}$

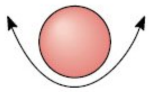
Long Horizontal Cylinder:

$$\overline{Nu_D} = \frac{\bar{h}L}{k} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/\Pr\right)^{9/16}\right]^{8/27}} \right\}^2 \text{ (Equations 9.34)}$$

Spheres:

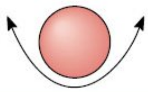
$$\overline{Nu_D} = \frac{\bar{h}L}{k} = \left\{ 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + \left(0.469/\Pr\right)^{9/16}\right]^{4/9}} \right\}^2 \text{ (Equations 9.35)}$$

4. Horizontal cylinder



	Equation 9.34	$Ra_D \lesssim 10^{12}$

5. Sphere



	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$