HEAT TRANSFER

CONVECTION

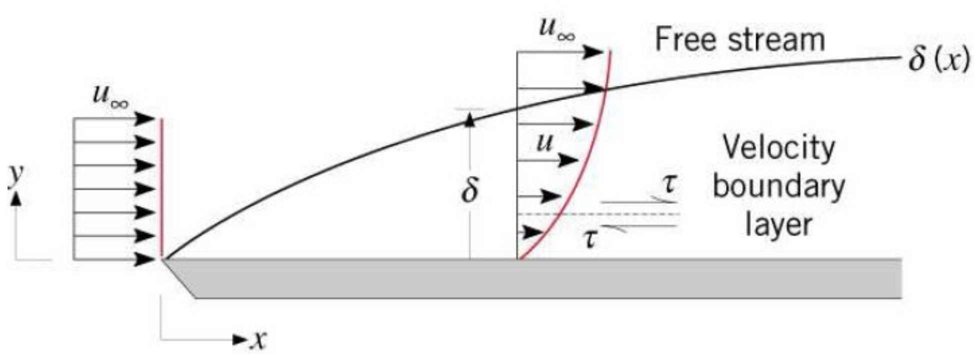
The Velocity Boundary Layer

When fluid flow over a solid surface, the first layer of the fluid sticks to the boundary of the surface – a condition called “no slip condition”.

This causes the flow of the fluid in the vicinity of the surface to retard.

As we move away from the wall the effect of the no slip condition gets smaller and smaller up to a point it is no longer felt by the fluid.

The layer affected by the no slip condition is termed boundary layer and δ is the boundary layer thickness measured in the y direction where $u = 0.99u_\infty$



The friction coefficient C_f is given by:

$$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2}$$

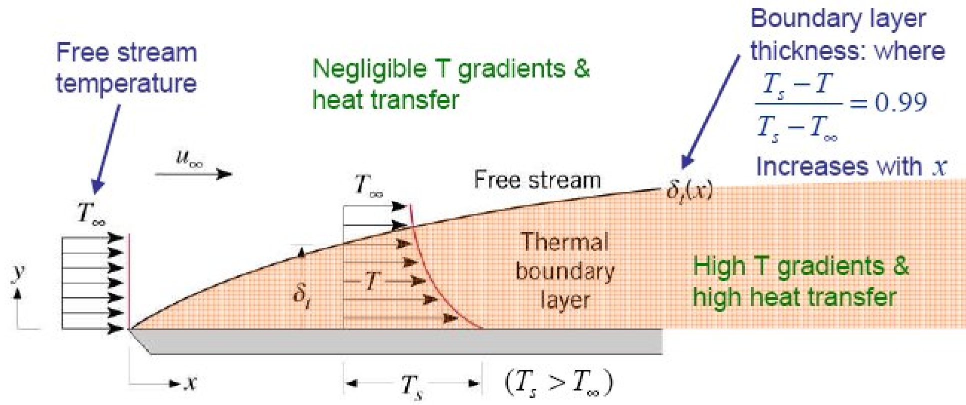
Thermal Boundary Layer

The same concept above can be applied for when a cold fluid flows over a how solid surface.

The first layer of the fluid gets its heat from the surface through pure conduction. It then gives its newly acquired energy to all the other fluid molecules that it comes in contact with. This is the convection mechanism in play.

As we move further and further away from the wall this effect is felt less until it no longer felt. The thermal boundary layer is where this effect is felt.

δ_t is the thermal boundary layer thickness in the y direction where the ratio at which point $\frac{T_s - T}{T_s - T_\infty} = 0.99$



Laminar and Turbulent Flow

Before we can analyze convection problems, we must establish if the flow is laminar or turbulent. Where laminar flow is characterized by smooth streamlines and highly ordered motion and turbulent flow is characterized by velocity fluctuations and highly disordered motion.

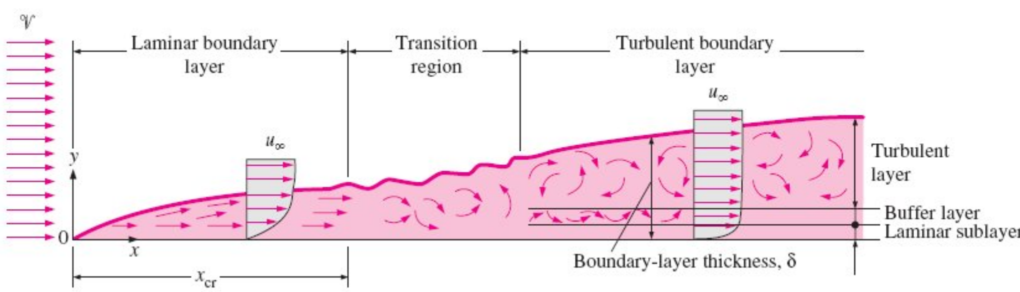
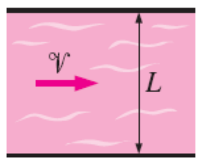


FIGURE 6–10
The development of the boundary layer for flow over a flat plate, and the different flow regimes.

The transition from laminar to turbulent does not occur suddenly but it occurs over a region when the flow fluctuates between laminar and turbulent flow.

The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, free stream velocity, surface temperature and type of fluid among other things. Nevertheless, it was discovered by a Mr. Osborn Reynolds that the flow regime depends mainly on the ratio of inertia force to viscous force in the fluid. This ratio is called the Reynolds



$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho \mathcal{V}^2/L}{\mu \mathcal{V}/L^2} = \frac{\rho \mathcal{V} L}{\mu} = \frac{\mathcal{V} L}{\nu}$$

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds Number**. The value of the critical Reynolds number is different for different geometries.

For flow over a flat plate, the generally accepted value of the critical Reynolds number is $\mathbf{Re_{cr} = 5 \times 10^5}$.

<u>Dimensionless Numbers in Heat transfer</u>

Dimensionless numbers are tools of heat transfer. They are not actually necessary to solve convection problems but the make it easier.

Key dimensionless numbers used in heat transfer are:

1. Reynolds number (Re)
2. Nusselt number (Nu)
3. Prandtl number (Pr)
4. Grashof number (Gr)
5. Rayleigh number (Ra)

Important values concerning these numbers are:

1. Viscosity is the fluid property due to cohesion and interaction between molecules which offer resistance to shear deformation of the fluid.

Dynamic viscosity (*μ*) is defined as the shear force per unit area required to drag one layer of fluid with unit velocity past another layer a unit distance away. Units are Ns/m^2
Can be found in tables

Kinematic viscosity (*v*) is the ratio of dynamic viscosity to density of the fluid. Units are m^2/s
Can be found in tables

2. **Thermal diffusivity (*α*)** is the ratio of a materials ability to conduct thermal energy to its ability to store thermal energy.

$$\alpha = \frac{k}{\rho C_p}, \text{ units are m}^2\text{/s}$$

Can be found in tables

3. **Coefficient of thermal expansion (*β*)** is the change in density of a substance as a function of temperature at constant pressure.

$$\beta = -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$$

For an ideal gas, $P = \rho RT$

$$\beta = \frac{1}{T}$$

For incompressible fluids and non ideal gases, β can be obtained from tables.

Reynolds Number

$$\mathrm{Re} = \frac{VL}{\upsilon}$$

Where V is the velocity of the fluid and L is the characteristic length of the object.

Reynolds number is the ratio of inertia force and viscous force. Viscous force maintains order in the boundary layer but inertia forces does the opposite. Hence large Reynolds number (large inertia forces) causes the flow to be turbulent. Laminar flow occurs when the Reynolds number is small.

Note:
The characteristic length for a

- Flat plate is the length of the plate in the direction of flow.
- Cylinder/sphere is the diameter of the cylinder/sphere

Nusselt Number

$$Nu = \frac{hL}{k}$$

The Nusselt number is the ratio of convection to conduction for a layer of fluid.

$$Nu = \frac{q''_{conv}}{q''_{cond}} = \frac{h\Delta T}{k\Delta T / L} = \frac{hL}{k}$$

Grashof Number

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}$$

This dimensionless number is mainly for natural convection.

It is the ratio of buoyancy forces to the viscous forces. The Grashof number plays the same role that the Reynolds number plays for forced convection. When buoyancy forces overcome viscous forces, the flow starts to become turbulent.

Prandtl Number

The Prandtl number is the ratio of kinematic viscosity and thermal diffusivity. It relates the thickness of the thermal and velocity boundary layer.

$$\mathrm{Pr} = \frac{C_p \mu}{k} = \frac{\upsilon}{\alpha}$$

Can be found in tables

Rayleigh Number

This number is just the product of the Grashof Number and the Prandtl number

$$Ra = Gr \cdot \mathrm{Pr}$$