THERMODYNAMICS

FIRST LAW OF THERMODYNAMICS – CLOSED SYSTEM

It has been pointed out that energy can neither be created nor destroyed; it can only change forms. This principle is the first law of thermodynamics or the conservation of energy principle.

During an interaction between a system and its surroundings, the amount of energy gained by the system must be exactly equal to the amount of energy lost by the surroundings.

Energy can cross the boundary of a closed system in the form of **heat** and **work**.

Heat Transfer

Heat transfer or just heat is defined as the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of temperature difference. Hence, if two systems are the same temperature, there can be no heat transfer.

A process during which there is no heat transfer is called an **adiabatic process**. A process can be adiabatic if both the system and its surroundings are at the same temperature **or** if the system is well insulated.

An adiabatic process is not necessarily an isothermal process!

The amount of heat transferred from state 1 to state 2 is demoted Q_{12} or just Q, it has units kJ.

$$q = \frac{Q}{m} \qquad \text{(kJ/kg)}$$

The heat transfer rate is given as

$$Q = \int_{t_1}^{t_2} \dot{Q} dt \qquad \text{(kJ)}$$

When the rate of heat transfer remains constant during a process

$$Q = \dot{Q}\Delta t \qquad \text{(kJ)}$$

Modes of Heat Transfer

There are three modes of heat transfer: conduction, convection and radiation.

Conduction heat transfer occurs in stationary solids, liquids and gasses. It occurs due to random molecular motion when atoms or molecules of the system collide. The heat transfer rate equation is called the **Fourier's law** given below.

$$\textbf{Fourier's law}: \quad \dot{Q}_{cond} = -k_t A \frac{dT}{dx} \quad (\text{W})$$

$$\dot{Q}_{cond} = k_t A \frac{\Delta T}{\Delta x} \qquad (\text{W})$$

Where Δx is the constant thickness of the system, ΔT is the temperature difference across the layer and A is the area perpendicular to the heat flow. Finally, k_t is called the **thermal conductivity** and it is a measure of a materials ability to transfer heat.

Convection heat transfer occurs between a solid surface and a moving fluid (liquid or gas). It occurs due to **random molecular motion** and **bulk fluid motion**. The first layer of fluid adjacent to the plate is stationary hence, heat transfer is due to conduction. Thereafter, it is due to hot fluid being carried away and replace by cooler fluid.

The heat transfer rate equation is called the Newton's law of cooling given below.

$$\textbf{Newton's law of cooling}: \quad \dot{Q}_{conv} = hA(T_s - T_f) \quad (\text{W})$$

Where h is the **convection heat transfer coefficient**, A is the surface area through which heat transfer takes place, T_s is the surface temperature and T_f is the bulk fluid temperature.

Radiation is the energy emitted by matter in the form of electromagnetic waves. Unlike conduction and convection, it does not require the presence of an intervening medium. In fact it is fastest through vacuum.

Solids, liquids and gasses can **emit, absorb**, reflect or transmit radiation.

The maximum rate of radiation that can be emitted by a surface at T_s is governed by the Stefan-Boltzmann law as

$$\dot{Q}_{rad} = \sigma A T_s^4 \qquad \text{(W)}$$

Where A is the area of the emitting surface and $\sigma = 5.67 \times 10^{-8}$ is the Stefan-Boltzmann constant. Only an idealized blackbody can emit this maximum amount of radiation at T_s. Real objects emit less and the rate of heat transfer is expressed:

$$\dot{Q}_{rad} = \varepsilon \sigma A T_s^4 \qquad \text{(W)}$$

Where ε is the emissivity and measures the ability of an object to emit radiation. It ranges between $0 \leq \varepsilon \leq 1$ where $\varepsilon = 1$ is a blackbody

Absorbed radiation also affects the energy levels in a system it is given by

$$\dot{Q}_{abs} = \alpha \dot{Q}_{inc} \qquad \text{(W)}$$

Where α is called absorptivity and it measure the ability of an object to absorb radiation.

In the special case of a small object of emissivity ε, a surface area A and absolute temperature T_s, is enclosed in a large surface at absolute temperature T_{surr}. The net rate of radiation heat transfer is given as:

$$\dot{Q}_{rad} = \varepsilon \sigma A \left(T_s^4 - T_{surr}^4 \right) \qquad \text{(W)}$$

Work

As mentioned, energy can cross a system boundary by heat transfer and work. Therefore, **if the energy crossing the boundary of a closed is not heat, it is work**. The amount of work done during a process between state 1 and state 2 is denoted by W_{12} or just W, it has units kJ.

The work done per unit mass is given as

$$w = \frac{W}{m} \qquad \text{(kJ/kg)}$$

The work done per unit time is called **power**.

Sign Convention for Heat and Work

Heat transfer into a system is **positive**.

Work done by the system is **positive**.

Heat transfer out of a system is **negative**.

Work done to the system is **negative**.

Electrical Work

When calculating rate of electrical work we use the expression:

$$\dot{W}_e = VI \qquad \text{(W)}$$

Where W_e is the electrical power, V is the potential difference and I is the current.

In the case where V and I varies with time, the electrical work done during a time interval Δt is expressed:

$$W_e = \int_1^2 VI dt \qquad \text{(kJ)}$$

When V and I are constant, expression above is reduced to

$$W_e = VI\Delta t \qquad \text{(kJ)}$$

Mechanical Forms of Work

We are familiar with the expression

$$W = Fs \qquad \text{(kJ)}$$

Where F is the force acting on a body and s is the distance traveled be the body.

If the force is not constant than

$$W = \int_1^2 Fds \qquad \text{(kJ)}$$

There are two requirements for work interaction between a system and its surroundings to exist:

1. There must be a force acting on the boundary and
2. The boundary must *move*.

Boundary Work

One form of mechanical work frequently encountered in practice is associated with the expansion or compression of a gas in a piston-cylinder device. During this process, part of the boundary moves back and forth. The expansion and compression work is called moving boundary work or just boundary work.

The boundary work is analysed for a **quasi equilibrium process** (or **quasi static process**), a process during which the system remains in equilibrium at all times. A quasi equilibrium process is achieved when the piston moves at low velocities. Under this condition, the system work output is maximum and the work input is minimum.
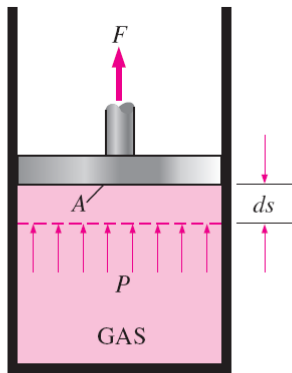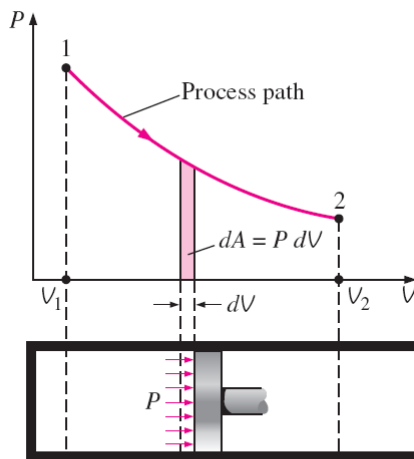

FIGURE 4–2

The differential boundary work done is

$$\delta W_b = Fds = PAds = PdV$$

The total boundary work done during the entire process is then

$$W_b = \int_1^2 PdV$$

Positive result indicates boundary work output (expansion).

Negative result indicates boundary work input (compression).

FIGURE 4–3

The area under the process curve on a P-V diagram represents the boundary work.

The area under the process curve on a P-V diagram is equal, in magnitude to the work done during a quasi-equilibrium expansion or compression of a closed system.
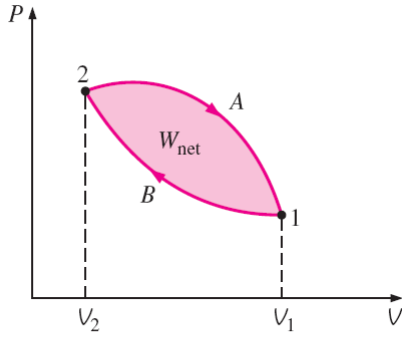
$$Area = A = \int_1^2 dA = \int_1^2 PdV = W_b$$

FIGURE 4–5

The net work done during a cycle is the difference between the work done by the system and the work done on the system.

Polytropic Process

During expansion and compression process of real gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. This is kind of process is called polytropic process.

Hence

$$P = CV^{-n}$$

Substituting into

$$W_b = \int_1^2 PdV ,$$

$$W_b = \int_1^2 PdV = \int_1^2 CV^{-n}dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

Since

$$C = P_1V_1^n = P_2V_2^n$$

For an ideal gas where

$$PV = mRT$$

The above equation can also be written

$$W_b = \frac{mR(T_2 - T_1)}{1-n}$$

Gravitational Work

Gravitational work is defined as work done by or against a gravitational force field. In a gravitational force field,

$$F = mg$$

Where **m** is the mass of the body and **g** is the acceleration of gravity. Then the work required to raise this body from level z_1 to z_2 is

$$W_g = \int_1^2 Fdz = mg\int_1^2 dz = mg(z_2 - z_1) \quad (kJ)$$

I.e. the change in **potential energy** where $z_2 - z_1$ is the vertical distance traveled.

Acceleration Work

The work associated with the change in velocity of a system is called acceleration work.

$$F = ma$$

$$a = \frac{dV}{dt}$$

$$F = m\frac{dV}{dt}n$$

Hence

$$W_g = \int_1^2 Fds = \int_1^2 \left(m\frac{dV}{dt}\right)(Vdt) = m\int VdV = \frac{1}{2}m(V_2^2 - V_1^2)$$

I.e. it is the change in **kinetic energy** of the body.

Shaft Work

A force F acting through a moment arm r generates a torque τ
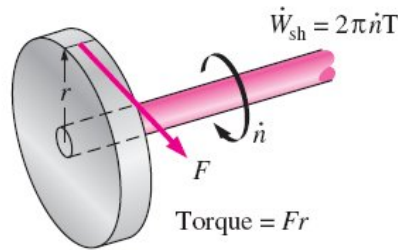

$$\dot{W}_{sh} = 2\pi \dot{n} T$$

FIGURE 2-28

Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

$$\tau = Fr$$

$$F = \frac{\tau}{r}$$

This force acts through a distance s where

$$s = (2\pi r)n$$

The shaft work is then

$$W_{sh} = Fs = \left(\frac{\tau}{r}\right)(2\pi rn) = 2\pi n\tau$$

The shaft power is given by

$$\dot{W}_{sh} = 2\pi \dot{n}\tau$$

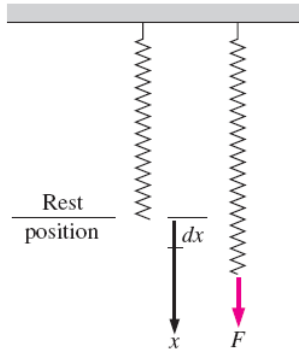Where n is the number of revolutions.

Spring work

Rest
position

dx

x F

FIGURE 2–30

Elongation of a spring under the influence of a force.

Work done by a spring is given as

$$W_{spring} = Fdx$$

Where

$$F = kx$$

And k is the spring constant given in kN/m.

Hence

$$W_{spring} = \frac{1}{2}k\left(x_2^2 - x_1^2\right) \qquad \text{(kJ)}$$

x_1 and x_2 are the initial and final displacement of the spring respectively

Continued…

The First Law of Thermodynamics

The first law of thermodynamics as stated previously, is the conservation of energy principle. In this section, we will use the first law of thermodynamics to relate **heat**, **Q**, **work**, **W** and, **total energy**, **E**.

Recall that total energy, E refers to the energy a system possesses.

$$E = U + KE + PE$$

Energy Balance

With regards to the first law of thermodynamics, *the net change (increase and decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process*.

Hence

$$E_{in} - E_{out} = \Delta E_{system} \qquad \text{ENERGY BALANCE}$$

Where

E_{in} = total energy entering the system

E_{out} = total energy leaving the system

E_{system} = change in the total energy of the system

Energy Change of a System, ΔE_{system} :

The energy change of a system is expressed as below

Energy change = Energy at final state – Energy at initial state

Or

$$\Delta E_{system} = E_{final} - E_{initial} = E_2 - E_1$$

Or simply

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

(Recall that E = U + KE + PE)

Where

$$\Delta U = m(u_2 - u_1)$$

$$\Delta KE = \frac{1}{2}m(V_2 - V_1)$$

$$\Delta PE = mg(z_2 - z_1)$$

Most systems encountered are stationary systems i.e. the do not change in velocity or elevation during a process. Hence the change in kinetic and potential energies are zero ($\Delta KE = \Delta PE = 0$)

Hence the energy balance is very often reduced to $\Delta E = \Delta U$.

Mechanisms for Energy Transfer, E_{in} and E_{out}:

Energy can be transferred to a system in three forms: *heat*, *work* and *mass flow*.

1. **Heat transfer, Q** Heat transfer to a system (heat gain) increases the energy of the system and heat transfer from the system (heat loss) decreases the energy of the system.

2. **Work, W**   Work transfer to a system (i.e. work done one a system) increase the energy of the system and work transfer from a system (i.e. work done by the system) decreases the energy of the system.

3. **Mass flow, m**   When mass enters a system, the energy of the system increases because mass carries energy with it. Likewise, when mass leaves the system it takes out some of the energy of the system.

Hence, the energy balance can be written as

$$E_{in} - E_{out} = \left(Q_{in} - Q_{out}\right) + \left(W_{in} - W_{out}\right) = \left(E_{mass,in} - E_{mass,out}\right) = \Delta E_{system}$$

The subscripts *in* and *out* denote quantities that enter and leave the system respectively. All six quantities on the right side of the equation represent "amounts", and thus they are positive quantities. The direction of any energy transfer is described by the subscript "in" and "out". Thus we do not need to adopt a formal sign convention.

On a per unit time basis the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{system}$$

On a per unit mass basis the energy balance is

$$e_{in} - e_{out} = e_{system}$$

For a closed system undergoing a cycle, the initial and final states are identical therefore

$$E_{system} = E_2 - E_1 = 0$$

- In terms of heat and work interactions

$$W_{net,out} = Q_{net,in}$$

When solving a problem that involves an unknown heat or work interaction, it is common practice to assume

$$Q - W = \Delta E$$

Where we assume there is a heat input into the system and work is done by the system (work output). When a negative answer is obtained for Q and W, it simply means the assumed direction is wrong and should be reversed.

Questions

1. The radiator of a steam heating system has a volume of 20 L and is filled with superheated vapour at 300 kPa and 250 °C. At this moment both the inlet and exit valves to the radiator are closed. Determine the amount of heat that will be transferred to the room when the steam pressure drops to 100 kPa. Also show the process on a *P-v* diagram with respect to saturation lines.

2. A 0.5 m3 rigid tank contains refrigerant-134a initially at 200 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 800 kPa. Determine (a) the mass of the refrigerant in the tank and (b) the amount of heat transferred. Also, show the process on a P-v diagram with respect to saturation lines.

3. A well insulated rigid tank contains 5 kg of a saturated liquid-vapour mixture of water at 100 kPa. Initially, three-quarters of the mass is in the liquid phase. An electric resistor placed in the tank is connected to a 240-V source and a current of 8 A flows through the resistor when the switch is turned on. Determine how long it will take to vaporize all the liquid in the tank. Also, show the process on a T-v diagram with respect to saturation lines.

4. An insulated tank is divided into two parts by a partition. One part of the tank contains 2.5 kg of compressed liquid water at 60 °C and 600 kPa while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the final temperature of the water and the volume of the tank for a pressure of 10 kPa.

5. A piston-cylinder device contains 5 kg of refrigerant-134a at 800 kPa and 60 °C. The refrigerant is now cooled at constant pressure until it exists as a liquid at 20 °C. Determine the amount of heat loss and show the process on a T-v diagram with respect to saturation lines.

6. A piston-cylinder device contains steam initially at 1 MPa, 350 °C, and 1.5 m^3. Steam is allowed to cool at constant pressure until it first starts condensing. Show the process on a T-v diagram with respect to saturation lines and determine (a) the mass of the steam, (b) the final temperature, and (c) the amount of heat transfer.

7. A piston cylinder device initially contains steam at 200 kPa. 200 °C and 0.5 m³. At this state, a linear spring is touching the piston but exerts no force on it. Heat is now slowly transferred to the steam, causing the pressure and the volume to rise to 500 kPa and 0.6m³, respectively. Show the process on a P-v diagram with respect to saturation lines and determine (a) the final temperature, (b) the work done by the steam, and (c) the total heat transferred.

8. A piston-cylinder device initially contains 0.5 m³ of saturated water vapour at 200 kPa. At this state, the piston is resting of a set of stops, and the mass of the piston is such tat a pressure of 300 kPa is required to move it. Heat is now slowly transferred to the steam until the volume doubles. Show the process on a P-v diagram with respect to saturation lines and determine (a) the final temperature, (b) the work done during this process, and (c) the total heat transfer.

9. Two rigid tanks are connected by a valve. Tank A contains 0.2 m³ of water at 400 kPa and 80 percent quality. Tank B contains 0.5 m³ of water at 200 kPa and 250 °C. The valve is now opened, and the two tanks eventually come to the same state. Determine the pressure and the amount to heat transfer when the system reaches thermal equilibrium with the surrounding at 25 °C.

Specific Heats

It takes different amounts of energy to raise the temperature of identical masses of different substances by one degree.

- We need about 4.5 kJ of energy to raise the temperature of 1 kg of iron to from 20 to 30 ºC

- It takes about 41.8 kJ of energy to raise the temperature of 1 kg liquid water from 20 to 30 ºC

It is desirable to have a property that will enable us to compare the energy storage capabilities of various substances. This property is the **specific heat**.

> The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

In thermodynamics, we are interested in the kinds of specific heats:

- Specific heat at constant volume (C_v)
- Specific heat at constant pressure (C_p)

The Constant Volume Heating Of a Gas

C_v can be viewed as the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Let a mass of gas (m) be heated at constant volume such that it rises from T_1 to T_2 and its pressure rises from P_1 to P_2. The heat received my the gas is

$$Q = mC_v(T_2 - T_1) \text{ ------------- Eq. 1}$$
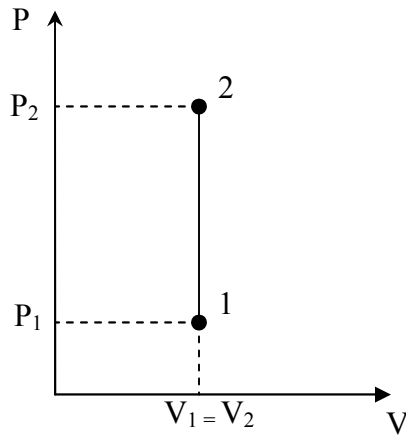
Applying the first law energy balance, assuming **KE = PE = 0** and **no work** is done ($\Delta v = 0$)

Therefore:

$$E_{in} - E_{out} = \Delta E_{system}$$

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

$$Q = \Delta U \quad \text{------------- Eq. 2}$$

Hence,

$$mC_v\left(T_2 - T_1\right) = \Delta U \quad \text{------------- Eq. 3}$$

Rewriting Eq. 3 produces

$$C_v\left(T_2 - T_1\right) = \Delta u$$

Therefore

$$C_v = \frac{\left(u_2 - u_1\right)}{\left(T_2 - T_1\right)} = \frac{\Delta u}{\Delta T}$$
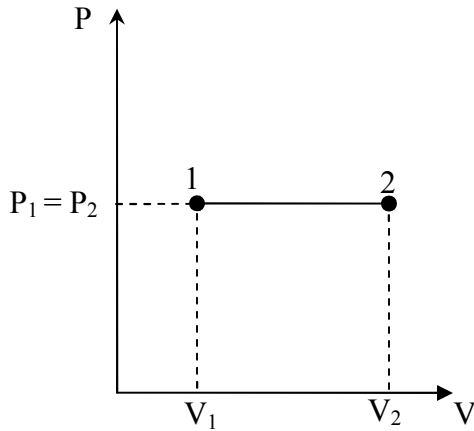
In differential form

$$C_v = \left\{\frac{du}{dT}\right\}_{v=cons\tan t}$$

The Constant Pressure Heating of a Gas

As before, for a constant pressure process, heat received by the gas is given by

$$Q = mC_p(T_2 - T_1) \text{ ------------- Eq. 4}$$

The process on the P-v diagram is show below

Work done during the process is given by the area under the curve. Therefore

$$W = P(V_2 - V_1)$$

$$W = P_2V_2 - P_1V_1 \text{ ------------- Eq. 5}$$

Applying the first law energy balance, assuming **KE = PE = 0** and **no work** is done ($\Delta v = 0$)

Therefore:

$$E_{in} - E_{out} = \Delta E_{system}$$

$$Q - W = \Delta U$$

$$Q = \Delta U + W$$

$$mC_p(T_2 - T_1) = (U_2 - U_1) + (P_2V_2 - P_1V_1)$$

$$mC_p(T_2 - T_1) = (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$mC_p(T_2 - T_1) = H_2 - H_1$$

$$C_p(T_2 - T_1) = h_2 - h_1 \text{ ------------- Eq. 6}$$

Rewriting Eq. 6 produces

$$C_p = \frac{(h_2 - h_1)}{(T_2 - T_1)} = \frac{\Delta h}{\Delta T}$$

In differential form

$$C_p = \left\{ \frac{dh}{dT} \right\}_{P = cons\,tan\,t}$$

Specific Heat Relations for Ideal Gases

A special relationship between C_p and C_v for ideal gases can be obtained by differentiating the relation

$$h = u + Pv$$
$$h = u + RT$$
$$dh = du + RdT$$

We know that $dh = C_p dT$ and $du = C_v dT$, therefore

$$C_p dT = C_v dT + RdT$$

Therefore,

$$C_p = C_v + R$$

Another important ideal gas relation is the specific heat ratio, γ which is given as

$$\gamma = \frac{C_p}{C_v}$$

$\gamma = 1.4$ for air at room temperature.

Internal Energy, Enthalpy, and Specific Heat Relations for Solids and Liquids

Specific Heat

A substance whose specific volume is constant is called incompressible substance – solids and liquids.

For incompressible substance (solids and liquids)

$$C_p = C_v = C$$

Internal Energy Changes

$$\Delta u = C_{av}(T_2 - T_1)$$

Where C_{av} is the average C value at average temperature.

Enthalpy Changes

By differentiating $h = u + Pv$, we obtain

$$dh = du + vdP + Pdv$$

Since v is constant then $dv = 0$, yielding

$$dh = du + vdP$$

Integrating this equation yields

$$\Delta h = \Delta u + v\Delta P = C_{av}\Delta T + v\Delta P$$

For solids, the term $v\Delta P$ is insignificant so

$$\Delta h = \Delta u = C_{av}\Delta T$$

For liquids, there are two possible cases

1) Constant pressure process (heater), ($\Delta P = 0$) therefore,

$$\Delta h = \Delta u = C_{av}\Delta T$$

2) Constant temperature process (heater), ($\Delta T = 0$) therefore,

$$\Delta h = v\Delta P$$

The Polytropic Process of a Gas

A gas undergoing a polytropic process expand or contracts according to

$$PV^n = C$$

For a process between state 1 and state 2 is then

$$P_1 V_1^n = P_2 V_2^n$$

We have also established that the work done during a polytropic process is given by:

$$W = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

Since $PV = mRT$

$$W = \frac{mR(T_2 - T_1)}{1 - n}$$

The Combination of the Polytropic Law $PV^n = C$ and the Ideal Gas Equation of a Perfect Gas

The law $PV^n = C$ will enable calculations to be made of the changes in pressure and volume which occur during a polytropic process. Combining this iwht the characteristic equation of a perfect gas will enable variations in temperature to be found.

Consider a polytropic process in which the state of a gas changes from P_1, V_1, T_1 to P_2, V_2, T_2.

By the polytropic law:

$$P_1 V_1^n = P_2 V_2^n \quad \text{------------- Eq. 1}$$

Hence:

$$\frac{P_1}{P_2} = \left\{ \frac{V_2}{V_1} \right\}^{n} \qquad \text{------------- Eq. 2}$$

By the ideal gas equation,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \qquad \text{------------- Eq. 3}$$

$$\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2} \qquad \text{------------- Eq. 4}$$

Hence, substituting Eq 2 into Eq 4,

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{n} \cdot \frac{V_1}{V_2}$$

Or,

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{(n-1)} \text{-------------- Eq. 5}$$

Also, from Eq 4

$$\frac{V_1}{V_2} = \left\{\frac{P_2}{P_1}\right\}^{1/n} \text{-------------- Eq. 6}$$

Substituting Eq 6 into Eq 4,

$$\frac{T_1}{T_2} = \left\{\frac{P_1}{P_2}\right\}^{\frac{(n-1)}{n}} \text{-------------- Eq. 7}$$

Combining Eq. 5 and Eq. 7

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{(n-1)} = \left\{\frac{P_1}{P_2}\right\}^{\frac{(n-1)}{n}}$$

Questions:

1) 2 kg of gas, occupying 0.7 m^3, had an original temperature of 15 ºC. If was then heated at constant volume until its temperature became 135 ºC. How much heat was transferred to the gas and what was its final pressure? Take C_v = 0.718 kJ/kg · K and R = 0.287 kJ/ kg · K.

2) A gas whose pressure, volume and temperature are 275 kPa, 0.09 m^3 and 185 ºC, respectively, has its state changed at constant pressure until its temperature becomes 15 ºC. How much heat is transferred form the gas and the work done on the gas during the process? Take R = 0.287 kJ/ kg · K. and C_p = 1.005 kJ/ kg · K.

3) A gas whose original pressure and temperature were 300 kPa and 25 ºC, respectively, is compressed according to the law $PV^{1.4}$ = C until its temperature becomes 180 ºC. Determine the pressure of the gas after it is compressed.

4) 0.675 kg of gas at 1.4 MPa and 280 ºC is expanded to four times the original volume according to the law $PV^{1.3}$ = C. Determine

   a) the original and final volume of the gas

   b) the final pressure of the gas

   c) the final temperature of the gas

   Take R = 0.287 kJ/ kg · K

5) 0.25 kg of air at pressure of 140 kPa occupies 0.15 m3 and form this condition it is compressed to 1.4 MPa according to the law $PV^{1.25}$ = C. Determine

   a) the change of internal energy of the air

   b) the work done on or by the air

   c) the heat received or rejected by the air

   Take C_p = 1.005 kJ/ kg · K and C_v = 0.718 kJ/kg · K

The Adiabatic Process of a Gas

When dealing with the general case of a polytropic expansion and compression, the process followed a law of the form $PV^n = C$.

Now the adiabatic process is a particular case of a polytropic process in which no heat is allowed to enter of leave the system

Consider an adiabatic process in which the state of a gas changes from P_1, V_1, T_1 to P_2, V_2, T_2.

Then,

$$\Delta U = mC_v(T_2 - T_1)$$

Also

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{mR(T_2 - T_1)}{1-\gamma}$$

Where gamma is known as the adiabatic index.

From the polytropic law,

$$P_1V_1^n = P_2V_2^n$$

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{(n-1)} = \left\{\frac{P_1}{P_2}\right\}^{\frac{(n-1)}{n}}$$

And the characteristic equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Applying the first law energy balance,

$$Q - W = \Delta U$$

But for an adiabatic process, $Q = 0$

$$-W = \Delta U$$

Hence,

$$\frac{mR(T_2 - T_1)}{1-\gamma} = mC_v(T_2 - T_1)$$

$$\frac{R}{1-\gamma} = C_v$$

Hence,

$$\frac{R}{C_v} = 1 - \gamma$$

$$\gamma = 1 - \frac{R}{C_v} = \frac{C_v - R}{C_v}$$

But $R = C_p - C_v$ therefore,

$$\gamma = \frac{C_p}{C_v}$$

γ for air is 1.4

Question:

A quantity of gas occupies a volume of 0.4 m³ at a pressure of 100 kPa and a temperature of 20 ºC. The gas is compressed isothermally to a pressure of 450 kPa and then expanded adiabatically to its initial volume. Determine, for this quantity of gas

        a) the heat transfer during the compression

        b) the change of internal energy during the expansion

        c) the mass of the gas

Take n = 1.4 and C_p = 1 kJ/ kg · K