HEAT TRANSFER

ANALYSIS OF HEAT EXCHANGER

The Effectiveness – NTU Method

This method is based on a dimensionless parameter called the heat transfer effectiveness, ε. Where

$$\varepsilon = \frac{q}{q_{\max}} = \frac{actual \quad heat \quad transfer \quad rate}{\max imum \quad possible \quad heat \quad tranfer \quad rate}$$

The actual heat transfer rate can be determined from

$$q = C_c\left(T_{c,out} - T_{c,in}\right)$$
$$q = C_h\left(T_{h,in} - T_{h,out}\right)$$

Where

$$C_h = \dot{m}_h C_{ph}$$
$$C_c = \dot{m}_c C_{pc}$$

The maximum possible heat transfer rate is given by

$$q_{\max} = C_{\min}\left(T_{h,in} - T_{c,in}\right)$$

Where C_{min} is the smaller of C_h and C_c and $\Delta T_{max} = T_{h,in} - T_{c,in}$

The determination of q_{max} requires the availability of the **inlet temperature** of the hot and cold fluids and their **mass flow rates**, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate q can be determined from

$$q = \varepsilon q_{\max}$$
$$q = \varepsilon C_{\min}\left(T_{h,in} - T_{c,in}\right)$$

To determine the effectiveness, ε we involve a dimensionless quantity called **number of transfer units** which is expressed as

$$NTU = \frac{UA}{C_{\min}}$$

Take note how, the larger the NTU, the larger the heat exchanger.

Another dimensionless quantity used is the **capacity ratio**, c which is expressed as

$$c = \frac{C_{\min}}{C_{\max}}$$

Hence, the effectiveness of a heat exchanger which depends on the **geometry** of the heat exchanger as well as the **flow arrangement** can be determined from the expressions given in the table below.

TABLE 13–4

Effectiveness relations for heat exchangers: $\text{NTU} = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\ \text{NTU}^{0.78}) - 1]\right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \frac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp\left\{-\frac{1}{c}[1 - \exp(-c\ \text{NTU})]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1 - \exp(-\text{NTU})$

The effectiveness, ε can also be determined graphically from Figure 13-26.

Subsequently, if we know ε, we can determine the NTU number from the expressions in the table below.

TABLE 13–5

NTU relations for heat exchangers $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$NTU = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$NTU = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$NTU = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow* (*single-pass*) C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with* $c = 0$	$NTU = -\ln(1-\varepsilon)$

Analysis of Heat Exchanger: Effectiveness – NTU Method

1. A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 $W/m^2 \cdot K$, determine the length of the heat exchanger required to achieve the desired heating.

2. Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 $W/m^2 \cdot °C$. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

3. Hot oil (Cp = 2200 J/kg · K) is to be cooled by water (Cp = 4180 J/kg · K) in a 2-shell-pass and 12-tube-pass heat exchanger. The tubes are thin-walled and are made of copper with a diameter of 1.8 cm. The length of each tube pass in the heat exchanger is 3 m, and the overall heat transfer coefficient is 340 $W/m^2 \cdot °C$. Water flows through the tubes at a total rate of 0.1 kg/s, and the oil through the shell at a rate of 0.2 kg/s. The water and the oil enter at temperatures 18°C and 160°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.