THERMODYNAMICS

ENTROPY CHANGE AND ISENTROPIC PROCESSES

Heat Transfer as the Area under a *T-S* Curve

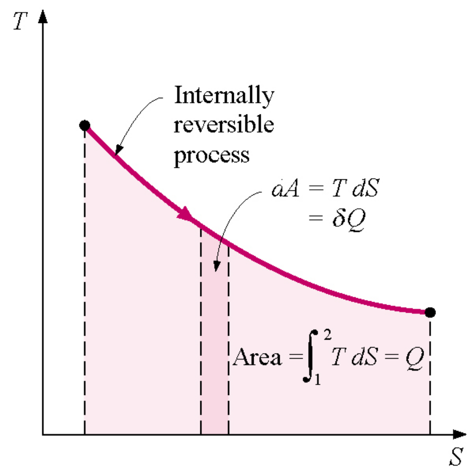
For the reversible process, the equation for *dS* implies that

$$dS = \frac{\delta Q_{net}}{T}$$

$$\delta Q_{net} = TdS$$

or the incremental heat transfer in a process is the product of the temperature and the differential of the entropy, the differential area under the process curve plotted on the *T-S* diagram.

In the above figure, the heat transfer in an internally reversible process is shown as the area under the process curve plotted on the T-S diagram.



$$Q_{net} = \int_1^2 TdS$$

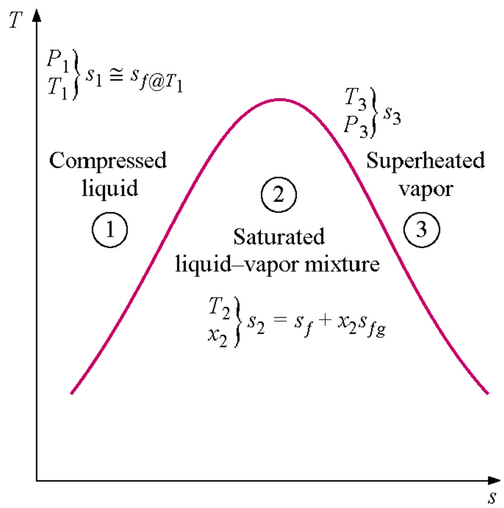
Entropy as a Property
Entropy is a property, and it can be expressed in terms of more familiar properties. On a unit mass basis we obtain the first *Tds* equation, or Gibbs equation, as

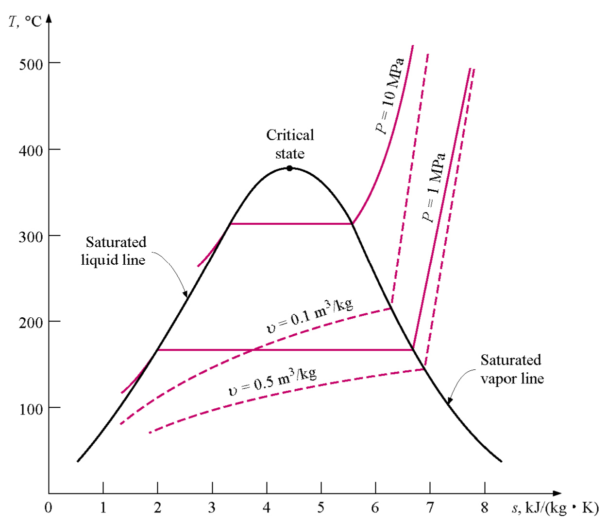
$$\boxed{Tds = du + Pdv}$$

Recall that the enthalpy is related to the internal energy by $h = u + Pv$. Using this relation in the above equation, the second *Tds* equation is

$$\boxed{T\,ds = dh - v\,dP}$$

The temperature-entropy and enthalpy-entropy diagrams for water are shown below.





Entropy Change and Isentropic Processes
The *entropy-change* and *isentropic relations* for a process can be summarized as follows:

1. ***Pure substances:***

Any process: $\Delta s = s_2 - s_1$ (kJ/kg·K)
Isentropic process: $s_2 = s_1$

2. Incompressible substances (Liquids and Solids):

$$ds = \frac{du}{T} + \frac{P}{T}dv$$

The change in internal energy and volume for an incompressible substance is

$$du = C\,dT$$
$$dv \cong 0$$

The entropy change now becomes

$$ds = \frac{C\,dT}{T} + 0$$
$$\Delta s = \int_1^2 \frac{C(T)\,dT}{T}$$

If the specific heat for the incompressible substance is constant ($C_v = C_p = C_{av}$), then the entropy change is

Any process:

$$s_2 - s_1 = C_{av} \ln\frac{T_2}{T_1}$$

Isentropic process:

$$T_2 = T_1$$

3. Ideal gases:

Assuming constant specific heats (approximate treatment):

$$s_2 - s_1 = C_{v,av} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \quad \text{(kJ/kg·K)}$$

$$s_2 - s_1 = C_{p,av} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad \text{(kJ/kg·K)}$$

Isentropic process:

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k}$$