

HEAT TRANSFER

TUTORIAL – CONDUCTION

- (1) Consider a medium in which the heat conduction equation is given in its simplest form as

$$k \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}$$

- (a) Is heat transfer steady or transient?
 - (b) Is heat transfer one-, two-, or three-dimensional?
 - (c) Is there heat generation in the medium?
 - (d) Is the thermal conductivity of the medium constant or variable?
- (2) Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r} \frac{d}{dr} \left(rk \frac{\partial T}{\partial r} \right) + \dot{\mathcal{Q}}_{gen} = 0$$

- (a) Is heat transfer steady or transient?
 - (b) Is heat transfer one-, two-, or three-dimensional?
 - (c) Is there heat generation in the medium?
 - (d) Is the thermal conductivity of the medium constant or variable?
- (3) Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

- (a) Is heat transfer steady or transient?
- (b) Is heat transfer one-, two-, or three-dimensional?
- (c) Is there heat generation in the medium?
- (d) Is the thermal conductivity of the medium constant or variable?

- (4) Consider a medium in which the heat conduction equation is given in its simplest form as

$$r \frac{\partial^2 T}{\partial r^2} + \frac{dT}{dr} = 0$$

- (a) Is heat transfer steady or transient?
 - (b) Is heat transfer one-, two-, or three-dimensional?
 - (c) Is there heat generation in the medium?
 - (d) Is the thermal conductivity of the medium constant or variable?
- (5) Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

- (a) Is heat transfer steady or transient?
 - (b) Is heat transfer one-, two-, or three-dimensional?
 - (c) Is there heat generation in the medium?
 - (d) Is the thermal conductivity of the medium constant or variable?
- (6) Consider a medium in which the heat conduction equation is given in its simplest form as

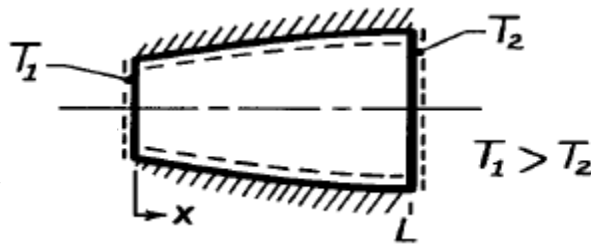
$$\frac{1}{r} \frac{d}{dr} \left(rk \frac{\partial T}{\partial r} \right) + \frac{d}{dz} \left(k \frac{\partial T}{\partial z} \right) + \dot{\phi}_{gen} = 0$$

- (a) Is heat transfer steady or transient?
- (b) Is heat transfer one-, two-, or three-dimensional?
- (c) Is there heat generation in the medium?
- (d) Is the thermal conductivity of the medium constant or variable?

- (7) Consider a medium in which the heat conduction equation is given in its simplest form as

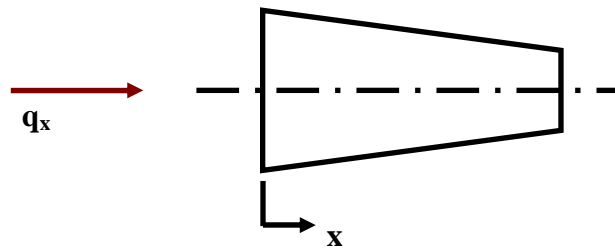
$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

- (a) Is heat transfer steady or transient?
- (b) Is heat transfer one-, two-, or three-dimensional?
- (c) Is there heat generation in the medium?
- (d) Is the thermal conductivity of the medium constant or variable?
- (8) Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown on the right. Assume constant properties and no internal heat generation, sketch the temperature distribution on T-x coordinates.

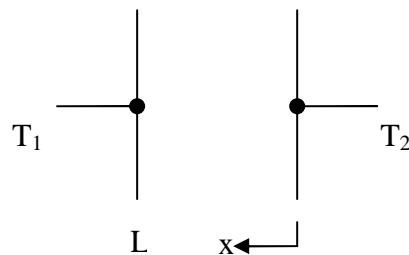


- (9) A hot water pipe with outside radius \$r_1\$ has a temperature \$T_1\$. A thick insulation applied to reduce the heat loss has an outer radius \$r_2\$ and temperature \$T_2\$. On T-r coordinates, sketch the temperature distribution in the insulation for one-dimensional, steady-state heat transfer with constant properties.

- (10) Assume steady-state, one-dimensional heat conduction through the symmetric shape shown. Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1-x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in Kelvin, and x in meters.

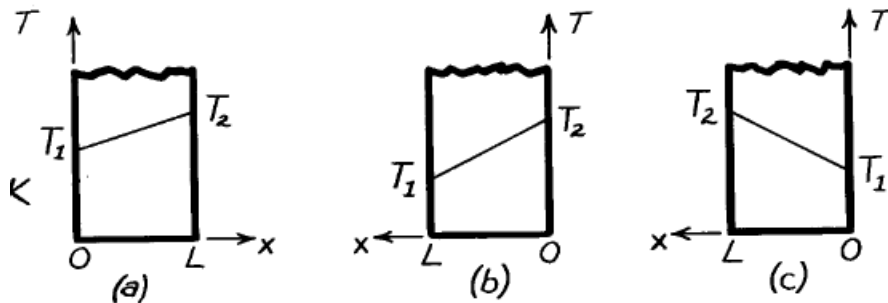


- (11) A cylinder of radius r_0 , length L , and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞ . At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at the fluid temperature.
- (12) One dimensional, steady-state conduction without heat generation occurs in the system shown. The thermal conductivity is 25 W/m.K and the thickness L is 0.5 m. Determine the unknown quantities for each case in the accompanying table and sketch the temperature distribution, indicating the direction of the heat flux.



Case	T_1	T_2	dT/dx (K/m)	q''_x (W/m ²)
1	400 K	300 K		
2	100 °C		-250	
3	80 °C		200	
4		-5 °C		
5	30 °C			

- (13) Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m.K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux and the temperature gradient for the coordinate systems shown.



- (14) Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m · K, and surface area $A = 15$ m². The two sides of the wall are maintained at constant temperatures of $T_1 = 120$ °C and $T_2 = 50$ °C, respectively. Determine (a) the variation of temperature within the wall and the value of temperature at $x = 0.1$ m and (b) the rate of heat conduction through the wall under steady conditions.

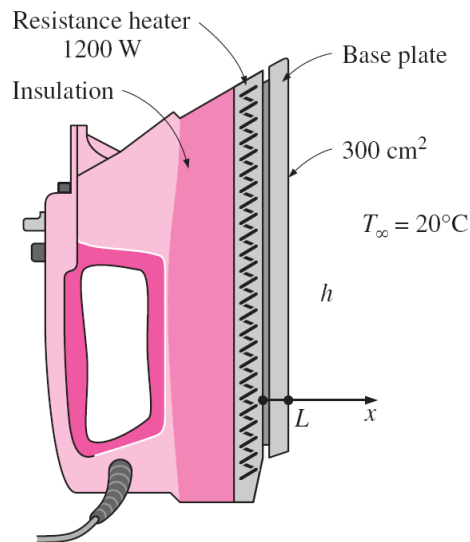
(Ans: 85°C, 6300W)

- (15) Consider steady one-dimensional heat conduction in a large plane wall of thickness L and constant thermal conductivity k with no heat generation. Obtain expressions for the variation of temperature within the wall for the following pair of boundary conditions:

$$-k \frac{dT(0)}{dx} = q_0'' = 40 \text{ W/cm}^2 \quad \text{and} \quad T(0) = T_0 = 15^\circ \text{C}$$

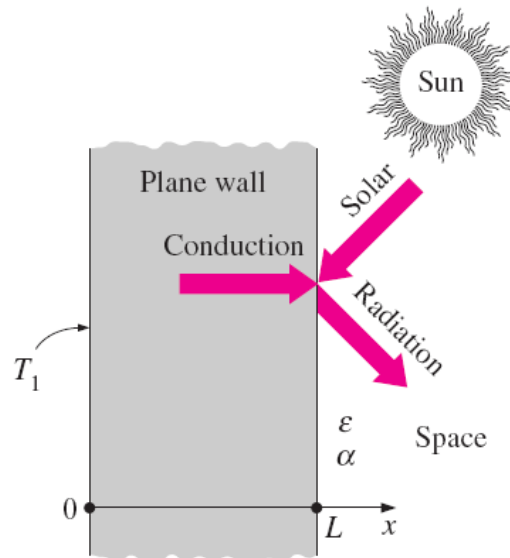
- (16) Consider the base plate of a 1200-W household iron that has a thickness of $L = 0.5 \text{ cm}$, base area of $A = 300 \text{ cm}^2$, and thermal conductivity of $k = 15 \text{ W/m} \cdot \text{K}$. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside, and the outer surface loses heat to the surroundings at $T = 20^\circ \text{C}$ by convection, as shown in the figure below. Taking the convection heat transfer coefficient to be $h = 80 \text{ W/m}^2 \cdot \text{K}$ and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces.

(Ans: 533°C , 520°C)



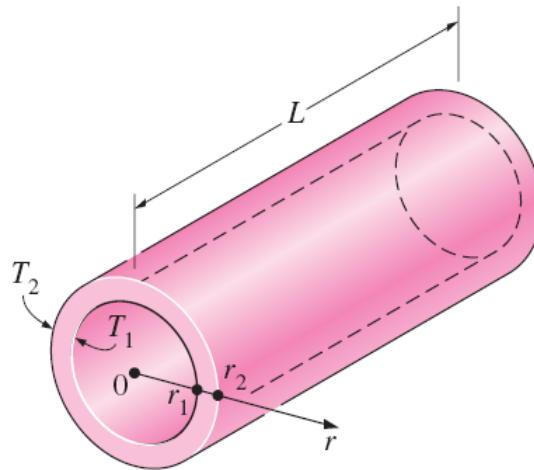
- (17) Consider a large plane wall of thickness $L = 0.06$ m and thermal conductivity $k = 1.2$ W/m \cdot $^{\circ}$ C in space. The wall is covered with white porcelain tiles that have an emissivity of $\epsilon = 0.85$ and a solar absorptivity of $\alpha = 0.26$, as shown in Figure 2–48. The inner surface of the wall is maintained at $T_1 = 300$ K at all times, while the outer surface is exposed to solar radiation that is incident at a rate of 800 W/m². The outer surface is also losing heat by radiation to deep space at 0 K. Determine the temperature of the outer surface of the wall and the rate of heat transfer through the wall when steady operating conditions are reached. What would your response be if no solar radiation was incident on the surface?

(Ans: 292.7 K, 146 W/m²)



- (18) Consider a steam pipe of length 20 m, inner radius of 6 cm, outer radius of 8 cm, and thermal conductivity of $20 \text{ W/m} \cdot ^\circ\text{C}$, as shown in the figure below. The inner and outer surfaces of the pipe are maintained at average temperatures of 150°C and 60°C , respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

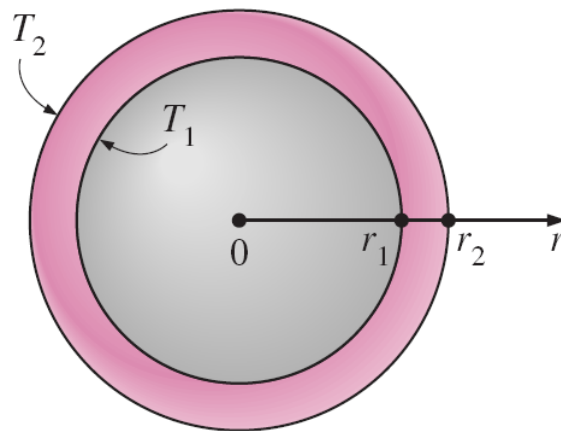
(Ans: 786 kW)



- (19) Consider a spherical container of inner radius of 8 cm, outer radius of 10 cm, and thermal conductivity of $45 \text{ W/m} \cdot ^\circ\text{C}$, as shown in Figure 2–52. The inner and outer surfaces of the container are maintained at constant temperatures of 200°C and 80°C , respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.

(Ans: 27,140

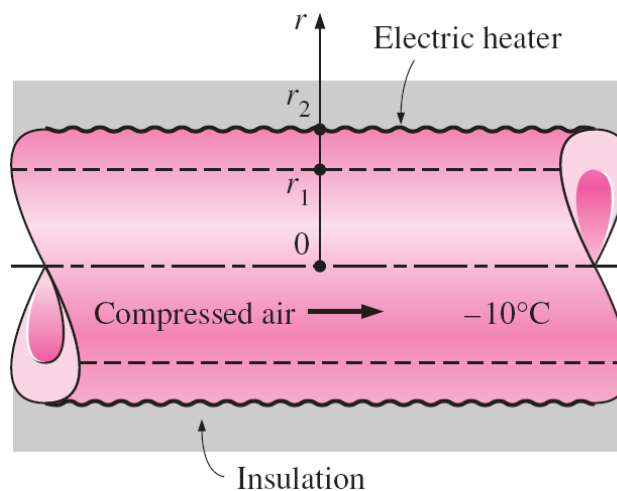
W)



- (20) When a long section of a compressed air line passes through the outdoors, it is observed that the moisture in the compressed air freezes in cold weather, disrupting and even completely blocking the air flow in the pipe. To avoid this problem, the outer surface of the pipe is wrapped with electric strip heaters and then insulated.

Consider a compressed air pipe of length 6 m, inner radius 3.7 cm, outer radius 4.0 cm, and thermal conductivity $14 \text{ W/m} \cdot ^\circ\text{C}$ equipped with a 300-W strip heater. Air is flowing through the pipe at an average temperature of 10°C , and the average convection heat transfer coefficient on the inner surface is $30 \text{ W/m}^2 \cdot ^\circ\text{C}$. Assuming 15 percent of the heat generated in the strip heater is lost through the insulation, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (b) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (c) evaluate the inner and outer surface temperatures of the pipe.

(Ans: (c) -3.91°C , -3.87°C)



- (21) In a food processing facility, a spherical container of inner radius 40 cm, outer radius 41 cm, and thermal conductivity $1.5 \text{ W/m} \cdot \text{K}$ is used to store hot water and to keep it at 100°C at all times. To accomplish this, the outer surface of the container is wrapped with a 500-W electric strip heater and then insulated. The temperature of the inner surface of the container is observed to be nearly 100°C at all times. Assuming 10 percent of the heat generated in the heater is lost through the insulation, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (b) obtain a relation for the variation of temperature in the container material by solving the differential equation, and (c) evaluate the outer surface temperature of the container. Also determine how much water at 100°C this tank can supply steadily if the cold water enters at 20°C

