Concepts of Stress and Strains
Stress-strain test is used to determine the mechanical behavior by applying a static load uniformly over a cross section or a surface of a member. The test is conducted at room temperature.

There are three principle ways a load can be applied:

1) Tension
Schematic illustration of how a tensile load produces and elongation and positive linear strain.

Dashed lines represent the shape before deformation, solid lines after deformation.

Figure 1(a)

2) Compression
Schematic representation of how a compressive load produces contraction and negative linear strain.

Figure 1(b)
3) Shear  
Schematic representation of shear loading

4) Torsion  
Schematic representation of torsional deformation.

There are four basic stress-strain tests based on the type of loading:
1. Tension test  
2. Compression test  
3. Shear test  
4. Torsional test
Tension Test
1. This is the most common of tests.
2. A specimen (Figure 2) is deformed, usually to fracture, by gradually increasing tensile load that is applied uniaxially along the long axis of a specimen.
3. The cross section is usually circular, but rectangular specimens are also used.
4. During testing, deformation is confined to the narrow center region, which has a uniform cross section along its length.
5. The standard diameter is approximately 12.8 mm (0.5 in.), whereas the reduced section length should be at least four times this diameter; 60 mm (2 1/4 in.) is common.
6. The specimen is mounted by its ends into the holding grips of the testing apparatus (Figure 3). The tensile testing machine is designed to elongate the specimen at a constant rate, and to continuously and simultaneously measure the instantaneous applied load (with a load cell) and the resulting elongations (using an extensometer).
7. The output of such a tensile test is recorded on a strip chart.
8. The test is usually plotted on an engineering stress – engineering strain (σ-ε) graph where:

   \[ \sigma = \frac{F}{A_0} \left( \frac{N}{m^2} \right) \quad \& \quad \varepsilon = \frac{\ell_i - \ell_0}{\ell_0} = \frac{\Delta \ell}{\ell_0} \]

   \( \sigma \) is engineering stress and \( \varepsilon \) is engineering strain.
   Strain is unitless but may be expressed as m/m or as a percentage.

Figure 2
A standard tensile specimen with circular cross section

Figure 3: Schematic representation of the apparatus used to conduct tensile stress–strain tests. The specimen is elongated by the moving crosshead; load cell and extensometer measure, respectively, the magnitude of the applied load and the elongation.
Compression Test
1. A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress.
2. By convention, a compressive force is taken to be negative, which yields a negative stress. Compressive strains are also negative.
3. Tensile tests are more common because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests.

Shear and Torsional Test
1. For tests performed using a pure shear force, the shear stress $\tau$ is computed according to where $F$ is the force imposed parallel to the upper and lower faces, each of which has an area of $A_0$.
2. The shear strain $\gamma$ is defined as the tangent of the strain angle $\theta$, as indicated in the Figure 1(c).
3. The units for shear stress and strain are the same as for their tensile counterparts.
   \[
   \tau = \frac{F}{A_0} \left( \frac{N}{m^2} \right) \quad \& \quad \gamma = \tan \theta
   \]
4. Torsion is a variation of pure shear, wherein a structural member is twisted in the manner of Figure 1(d).

Tension test carried out shows that deformation can be classified as elastic deformation or plastic deformation. Plotting the stress and strains of the test on a graph would reveal:
Elastic Deformation

Elastic deformation is temporary deformation, as soon as the load is removed, the member would return to its original form.

Stress-Strain Behavior

1. Hooke’s law gives the relationship between stress and strain for elastic deformation under tensile and compressive loads. Where:

   \[ \sigma = E \varepsilon \]

   Hooke’s law

   E (GPa or psi) is called modulus of elasticity or Young’s modulus.

Refer to Table 1 for Modulus of elasticity values for several metals

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity</th>
<th>Shear Modulus</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa 10^9 psi</td>
<td>GPa 10^9 psi</td>
<td></td>
</tr>
<tr>
<td>Metal Alloys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tungsten</td>
<td>407 59</td>
<td>160 23.2</td>
<td>0.28</td>
</tr>
<tr>
<td>Steel</td>
<td>207 30</td>
<td>83 12.0</td>
<td>0.30</td>
</tr>
<tr>
<td>Nickel</td>
<td>207 30</td>
<td>76 11.0</td>
<td>0.31</td>
</tr>
<tr>
<td>Titanium</td>
<td>107 15.5</td>
<td>45 6.5</td>
<td>0.34</td>
</tr>
<tr>
<td>Copper</td>
<td>110 16</td>
<td>46 6.7</td>
<td>0.34</td>
</tr>
<tr>
<td>Brass</td>
<td>97 14</td>
<td>37 5.4</td>
<td>0.34</td>
</tr>
<tr>
<td>Aluminum</td>
<td>69 10</td>
<td>25 3.6</td>
<td>0.33</td>
</tr>
<tr>
<td>Magnesium</td>
<td>45 6.5</td>
<td>17 2.5</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 1: Room-temperature Elastic and Shear Moduli, and Poisson’s for Various Materials

2. Elastic deformation exhibit proportional stress and strain relationship as in Figure 4.
3. The slope of the graph corresponds to the modulus of elasticity, E.
4. Modulus of elasticity may be thought of as stiffness (resistant to plastic deformation). The greater the modulus of elasticity, the stiffer the material.
5. Metals and ceramic have comparable modulus of elasticity but for polymer it is much lower.
6. Modulus of elasticity also reduces at higher temperatures (materials loose their stiffness at higher temperatures).
7. In the case of compressive forces, at low stress levels, the modulus of elasticity is about the same.

8. For shear and torsional forces, the shear stress and shear strain and related through the shear modulus, G.

\[ \tau = G\gamma \]

Example:
A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?
Poisson’s Ratio
A longitudinal elastic deformation of a metal produces an accompanying lateral dimensional change. The ratio of these deformations is called the Poisson’s Ratio, \( v \):

\[
\nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{longitudinal}}} = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}
\]

Figure 6
Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

Plastic Deformation
1. For most metallic materials, elastic deformation persists only to strains of about 0.005.
2. Deformation beyond this point will not cause the material to return to its original form even after the load is removed.
3. The deformation is permanent and it is called plastic deformation.
4. The stress and strain relationship is no longer proportional hence Hooke’s law is no longer valid.
5. The transition from plastic to elastic occurs gradually as seen on the curve of the graph (Figure 7).
Mechanical Properties

We will consider:

1. Yielding and yield strength
2. Tensile strength
3. Ductility
4. Toughness
5. Hardness

1. Yielding and yield strength
   (i) Most engineering structures are designed to ensure only elastic deformation will result under load. Plastic deformation or permanent damage may not stop the structure from functioning appropriately.
   (ii) Therefore, it is important to know when the plastic deformation begins (phenomenon called yielding).
   (iii) The point of yielding is the point when the stress-strain curve is no longer linear. This point is also called the proportional limit and it is indicated by point P on Figure 7.
   (iv) Since this exact point maybe difficult to determine precisely, yielding point is usually determined by drawing a line parallel to elastic portion of the stress-strain curve that has been offset by 0.002.
   (v) The stress at the point where the parallel line meets the strain-curve is the called the yield strength, \( \sigma_y \) (units MPa or psi).

![Figure 7](image)

Figure 7
(a) Typical stress–strain behavior for a metal showing elastic and plastic deformations, the proportional limit \( P \), and the yield strength \( \sigma_y \), as determined using the 0.002 strain offset method.
2. Tensile strength

(i) After yielding, the stress necessary to continue plastic deformation in metals increases to maximum point M (Figure 8), and the decreases until fracture, Point F.

(ii) The **tensile strength**, TS (MPa or psi) is the stress at the maximum on the engineering stress-strain curve. This is the maximum stress that can be sustained by a structure in tension, if this stress is applied and maintained, fracture will result.

(iii) At this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at this neck (Figure 8).

(iv) For engineering design purpose, the yield strength is specified. If tensile strength is used too much plastic deformation would take place. Fracture strength is not normally specified.

![Figure 8: Schematic representation of tensile stress-strain behavior for brittle and ductile materials loaded to fracture.](image)
3. Ductility

(i) Ductility is a measure of the degree of plastic deformation that has been sustained at fracture.

(ii) A material that experiences very little or no plastic deformation upon fracture is termed brittle.

(iii) Ductility can be expressed as percent elongation (% EL) or percent reduction in area (% RA).

\[
\% EL = \left( \frac{l_f - l_0}{l_0} \right) \times 100
\]

Where \( l_f \) is the fracture length and \( l_0 \) is the gauge length.

\[
\% RA = \left( \frac{A_0 - A_f}{A_0} \right) \times 100
\]

Where \( A_f \) is cross-sectional area at the fracture and \( A_0 \) is the original cross-sectional area.

Figure 9: schematic representation of tensile stress-strain behavior for brittle and ductile materials loaded to fracture
4. Toughness
   
   (i) Toughness is the measure of the ability of a metal to absorb energy up to fracture.

   (ii) The key to toughness is a good combination of strength and ductility. A material with high strength and high ductility will have more toughness than a material with low strength and high ductility.

   (iii) It is the area under the stress-strain curve up to the point of fracture.

(iv) Fracture toughness is an indication of the amount of stress required to propagate a pre-existing flaw. It is an important property since flaws cannot be avoided completely the processing, fabrication, or service of a material/component. Flaws may appear as cracks, voids, metallurgical inclusions, weld defects, design discontinuities, or some combination thereof. It is used to evaluate the ability of a component containing a flaw to resist fracture.

5. Hardness
   
   (i) Hardness is a measure a materials resistance to localized plastic deformation (e.g., a small dent or a scratch).

   (ii) To measure hardness, a small indenter is forced into the surface of a material to be tested, under controlled condition of load and rate of application. The depth or size of the resulting indentation is measured, which in turn is related to a hardness number. The softer the material, the larger and deeper the indentation, and the lower the hardness index number.

   (iii) Hardness test is performed more frequently than any other mechanical test because:

       (a) They are simple and inexpensive

       (b) The test is non destructive

       (c) Other mechanical properties often may be estimated from hardness data such as tensile strength.
There are several common hardness tests, with its own hardness index:

a. Rockwell hardness test
b. Brinell hardness test

Rockwell hardness test

a. This is the most common method of testing hardness.
b. Indenters used include spherical and hardened steel balls – Diameter, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ in. (1.588, 3.175, 6.350, 12.70 mm) and a conical diamond indenter, which is used for the hardest materials.
c. The hardness is determined by the difference in **depth of penetration** resulting from the application of an initial minor load followed by a large major load.
d. There are two types of hardness test Rockwell and superficial Rockwell.

Rockwell – minor load is 10 kg and major load is 60, 100 and 150 kg

<table>
<thead>
<tr>
<th>Table 7.5a</th>
<th>Rockwell Hardness Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scale</strong></td>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>A</td>
<td>Diamond</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{1}{16}$ in. ball</td>
</tr>
<tr>
<td>C</td>
<td>Diamond</td>
</tr>
<tr>
<td>D</td>
<td>Diamond</td>
</tr>
<tr>
<td>E</td>
<td>$\frac{1}{8}$ in. ball</td>
</tr>
<tr>
<td>F</td>
<td>$\frac{1}{16}$ in. ball</td>
</tr>
<tr>
<td>G</td>
<td>$\frac{1}{16}$ in. ball</td>
</tr>
<tr>
<td>H</td>
<td>$\frac{1}{6}$ in. ball</td>
</tr>
<tr>
<td>K</td>
<td>$\frac{1}{8}$ in. ball</td>
</tr>
</tbody>
</table>

80 HRB = Rockwell hardness of 80 on the B Rockwell scale
60 HRF = Rockwell hardness of 60 on the F Rockwell scale
Superficial Rockwell – minor load is 3 kg and major load is 15, 30 and 45 kg

**Table 7.5b  Superficial Rockwell Hardness Scales**

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15N</td>
<td>Diamond</td>
<td>15</td>
</tr>
<tr>
<td>30N</td>
<td>Diamond</td>
<td>30</td>
</tr>
<tr>
<td>45N</td>
<td>Diamond</td>
<td>45</td>
</tr>
<tr>
<td>15T</td>
<td>5/32 in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30T</td>
<td>3/16 in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45T</td>
<td>1/8 in. ball</td>
<td>45</td>
</tr>
<tr>
<td>15W</td>
<td>5/32 in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30W</td>
<td>3/16 in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45W</td>
<td>1/8 in. ball</td>
<td>45</td>
</tr>
</tbody>
</table>

Scales are identified by a 15, 30 and 45 according to load followed by a letters N, T, W, X or Y according to indenter.

60 HR30W = Rockwell hardness of 60 on the 30W superficial Rockwell scale.

(vi) Brinell hardness test

a. In the Brinell test, a hard, spherical indenter is forced into the surface of the metal to be tested.

b. The diameter of the hardened steel indenter is 10.00 mm (0.394 in)

c. Standard loads of 500 and 3000 kg in 500 kg increments is used.

d. The load is maintained constant for a specified time (between 10 and 30 s)

e. The Brinell hardness number, HB, is a function of the magnitude of the load and the diameter of the resulting indentation.

(vii) Knoop and Vickers micro-indentation hardness test

a. For each test, a very small diamond indenter having pyramidal geometry is forced into the surface of the specimen.

b. Applied load are much smaller compared to the other two tests (between 1 and 1000 g).

c. The resulting indentation is observed under a microscope and measured.

d. The diameter of the hardened steel indenter is 10.00 mm (0.394 in)

e. The hardness number is a function of the magnitude of the load and the mean diagonal diameter of the resulting indentation.
Tutorial

1. A specimen of copper having a rectangular cross section 15.2 mm x 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.

2. A cylindrical specimen of a nickel alloy having an elastic modulus of 205 GPa and an original diameter of 10.2 mm will experience only elastic deformation when a tensile load of 8900 N is applied. Compute the maximum length of specimen before deformation if the maximum allowable elongation is 0.25 mm.

3. An aluminum bar 125 mm long and having a square cross section 16.5 mm on an edge is pulled in tension with a load of 66,700 N, and experiences an elongation of 0.43 mm. Assuming that the deformation is entirely elastic, calculate the modulus of elasticity of the aluminum.

4. For a brass alloy, the stress at which plastic deformation begins is 345 MPa and the modulus of elasticity is 103 GPa.
   a. What is the maximum load that may be applied to a specimen with cross-sectional area of 130 mm² without plastic deformation?
   b. If the original specimen length is 76 mm, what is the maximum length to which it may be stretched without causing plastic deformation?

5. What are elastic deformation and plastic deformation?

6. A 10 mm diameter, 500mm long stainless steel 309 rod is subjected to a tensile load of 30,000 N, what is the engineering stress experienced by the rod? Show that the rod will return to its original length after the tensile load is removed. Then determine resultant elongation of the rod. The stainless steel 309 yield strength is 290 MPa and the modulus of elasticity is 200 GPa.

7. A cylindrical specimen of some metal alloy 0.4 in. diameter is stressed elastically in tension. A force of 3370 lbf produces a reduction in specimen diameter of 2.8 × 10⁻⁴ in. Compute Poisson’s ratio for this material of its elastic modulus is 14.5 × 10⁶ psi.

8. Consider a cylindrical specimen of a hypothetical metal alloy that has a diameter of 10.0 mm (0.39 in.). A tensile force of 1500 N (340 lbf) produces an elastic deformation of this alloy, given that Poisson’s ratio is 0.35.
9. The following engineering tensile stress-strain data were obtained for a 0.2% C plain-carbon steel.

<table>
<thead>
<tr>
<th>Engineering Stress (ksi)</th>
<th>Engineering Strain (%)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>0.1</td>
</tr>
<tr>
<td>55</td>
<td>0.2</td>
</tr>
<tr>
<td>60</td>
<td>0.5</td>
</tr>
<tr>
<td>68</td>
<td>1.0</td>
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<tr>
<td>72</td>
<td>2.0</td>
</tr>
<tr>
<td>74</td>
<td>4.0</td>
</tr>
<tr>
<td>75</td>
<td>6.0</td>
</tr>
<tr>
<td>76</td>
<td>8.0</td>
</tr>
<tr>
<td>75</td>
<td>10.0</td>
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<tr>
<td>73</td>
<td>12.0</td>
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<tr>
<td>69</td>
<td>14.0</td>
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<tr>
<td>65</td>
<td>16.0</td>
</tr>
<tr>
<td>56</td>
<td>18.0</td>
</tr>
<tr>
<td>51</td>
<td>19.0 (fracture)</td>
</tr>
</tbody>
</table>

(i) Plot the engineering stress-strain curve, with stress in units of ksi and strain expressed as a %.

(ii) Determine the ultimate tensile strength of the alloy.

(iii) Determine the percent elongation at fracture.
Assignment II

A cylindrical specimen of aluminum having a diameter of 0.505 in. (12.8 mm) and a
gauge length of 2.000 in. (50.800 mm) is pulled in tension. Use the load–elongation
characteristics tabulated below to complete problems (a) through (e).

a. Plot the data as engineering stress versus engineering strain.
b. Compute the modulus of elasticity
c. Determine the yield strength at a strain offset of 0.002.
d. Determine the tensile strength of this alloy.
e. What is the approximate ductility, in percent elongation?

<table>
<thead>
<tr>
<th>Load</th>
<th>Length</th>
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</thead>
<tbody>
<tr>
<td>lb</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1,650</td>
<td>7,330</td>
</tr>
<tr>
<td>3,400</td>
<td>15,100</td>
</tr>
<tr>
<td>5,200</td>
<td>23,100</td>
</tr>
<tr>
<td>6,850</td>
<td>30,400</td>
</tr>
<tr>
<td>7,750</td>
<td>34,400</td>
</tr>
<tr>
<td>8,650</td>
<td>38,400</td>
</tr>
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<td>9,300</td>
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<td>10,100</td>
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<td>47,300</td>
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<td>10,400</td>
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<tr>
<td>10,100</td>
<td>44,800</td>
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<tr>
<td>9,600</td>
<td>42,600</td>
</tr>
<tr>
<td>8,200</td>
<td>36,400</td>
</tr>
</tbody>
</table>

Fracture

Students must use graph paper to plot the graph

Due date: 10th March 2009

Lateness without suitable reason will not be tolerated.