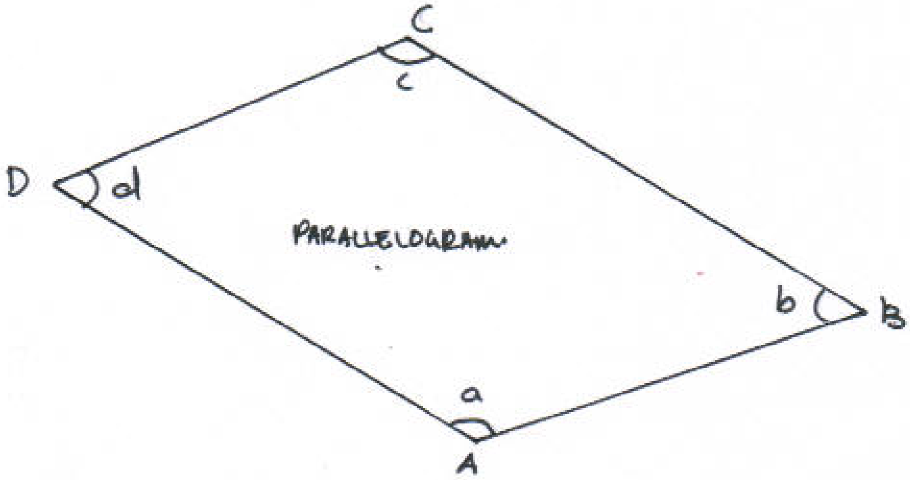
GEOMETRY OF A PARALLELOGRAM:



SUM OF ANGLES:

$$a + b + c + d = 360^\circ$$

OPPOSITE ANGLES:

$$a = c$$
$$b = d$$

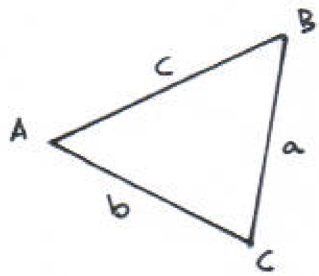
OPPOSITE SIDES:

$$AB = CD$$
$$AD = CB$$

PARALLEL SIDES:

$$AB \parallel CD$$
$$AD \parallel CB$$

OBLIQUE TRIANGLES:



THE LAW OF COSINES:

$$c^2 = \sqrt{a^2 + b^2 - 2ab\cos C}$$

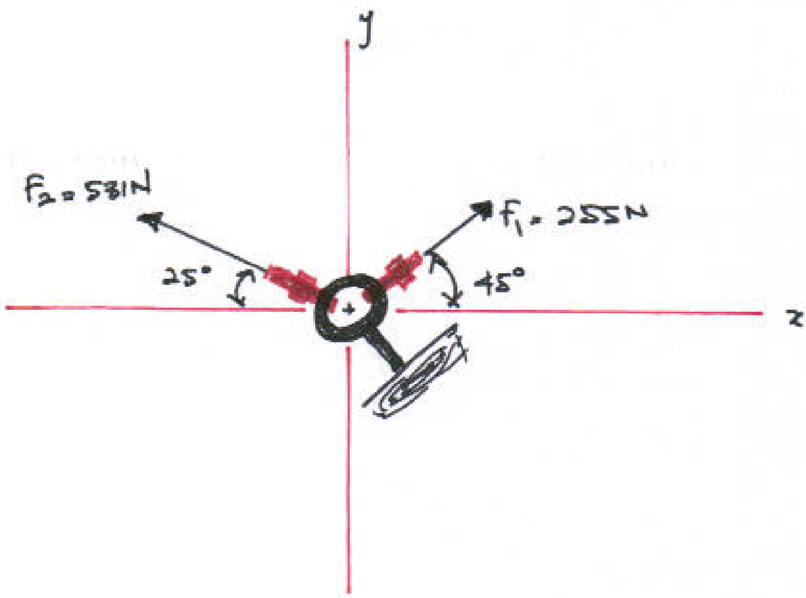
THE LAW OF SINES:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

ENGINEERING MECHANICS – STATICS

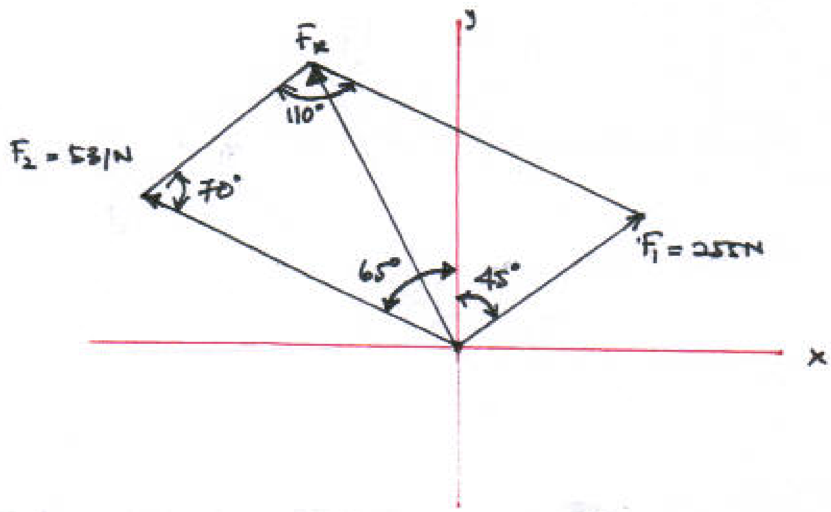
PARALLELOGRAM LAW

1) DETERMINE THE MAGNITUDE OF THE RESULTANT FORCE $\vec{F}_R = \vec{F}_1 + \vec{F}_2$ AND ITS DIRECTION, MEASURED COUNTERCLOCKWISE FROM THE POSITIVE X-AXIS.

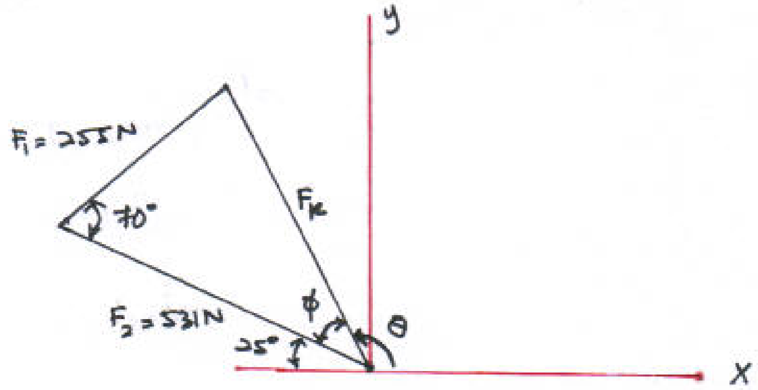


SOLUTION:

- SKETCH THE PARALLELOGRAM OF FORCES $\vec{F}_1$ & $\vec{F}_2$:



- SKETCH THE TRIANGLE FROM THE PARALLELOGRAM:



- USING COSINE LAW TO SOLVE F_R :

$$F_R = \sqrt{F_1^2 + F_2^2 - 2(F_1)(F_2)\cos 70°}$$

$$= \sqrt{285^2 + 531^2 - 2(255)(531)\cos 70°}$$

$$\underline{\underline{F_R = 504N}}$$

- USING SINE LAW TO SOLVE ϕ :

$$\frac{F_R}{\sin 70} = \frac{F_1}{\sin\phi}$$

$$\frac{504}{\sin 70} = \frac{255}{\sin\phi}$$

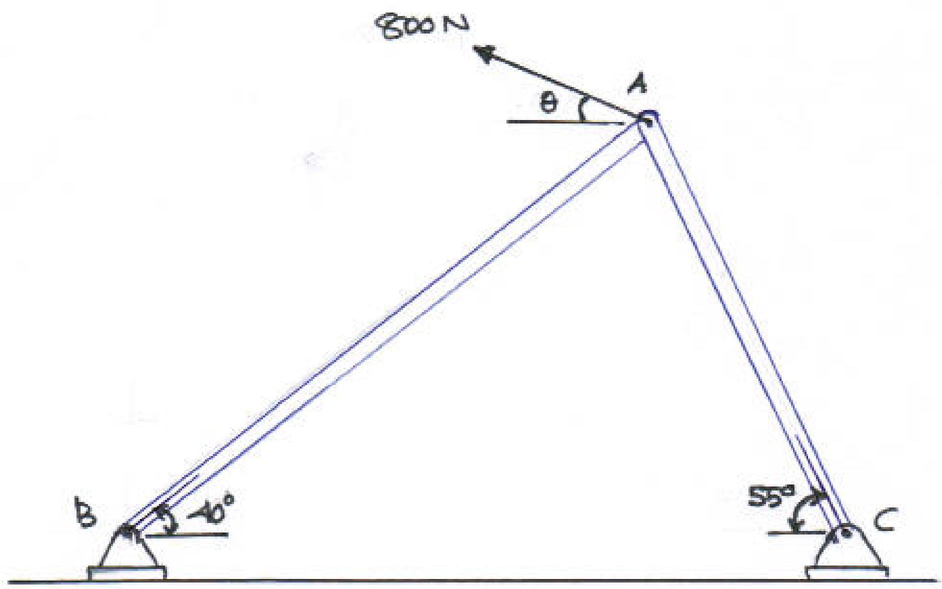
$$\therefore \phi = 28°$$

HENCE :

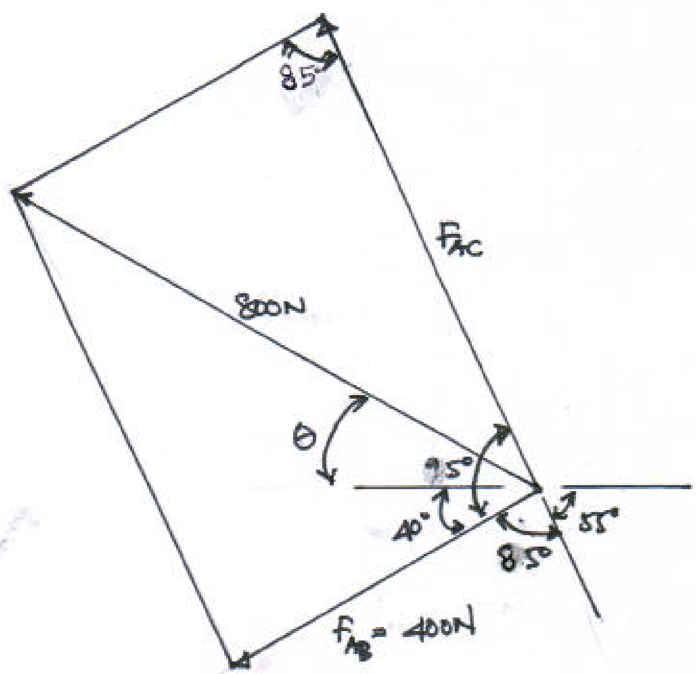
$$\theta = 180° - 25° - 28°$$

$$\underline{\underline{\theta = 127°}}$$

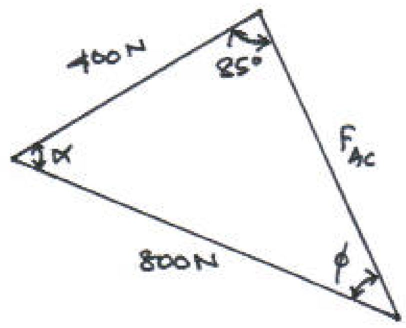
2) THE 800 N FORCE ACTING ON THE FRAME IS TO BE RESOLVED INTO TWO COMPONENTS ACTING ALONG THE AXIS OF THE STRUTS AB AND AC. IF THE COMPONENT OF FORCE ALONG AB IS REQUIRED TO BE 400N DIRECTED FROM A TO B DETERMINE THE MAGNITUDE OF FORCE ACTING ALONG AC AND THE ANGLE θ OF THE 800 N FORCE



- SKETCH THE PARALLELOGRAM:



SKETCH THE TRIANGLE:



USING SINE LAW TO SOLVE ϕ:

$$\frac{800}{\sin 85} = \frac{400}{\sin \phi}$$

$$\therefore \phi = 30°$$

$$\therefore \alpha = 180 - 85 - 30$$

$$= 65°$$

USING SINE LAW TO SOLVE F_{AC}:

$$\frac{F_{AC}}{\sin 65°} = \frac{800}{\sin 85°}$$

$$\therefore \underline{\underline{F_{AC} = 728 \text{ N}}}$$

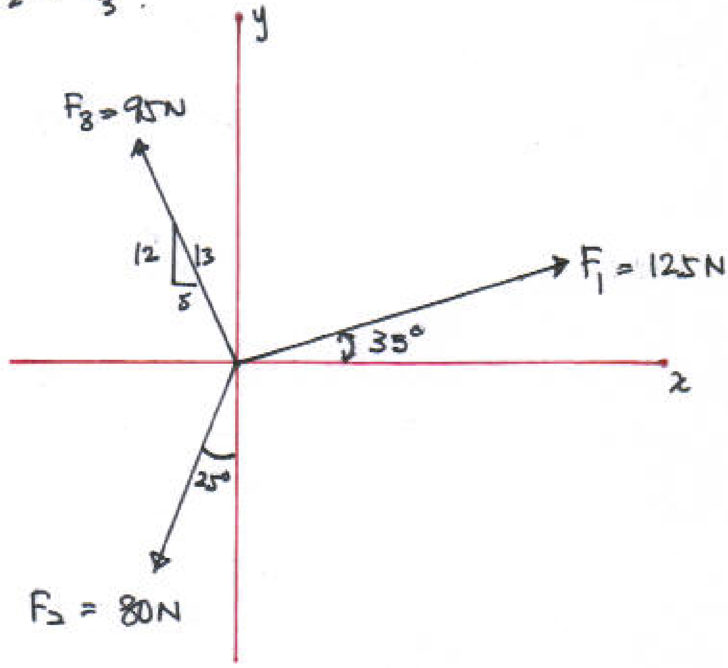
SOLVING θ:

$$\theta = 95° - 40° - \phi$$

$$= 95° - 40° - 30°$$

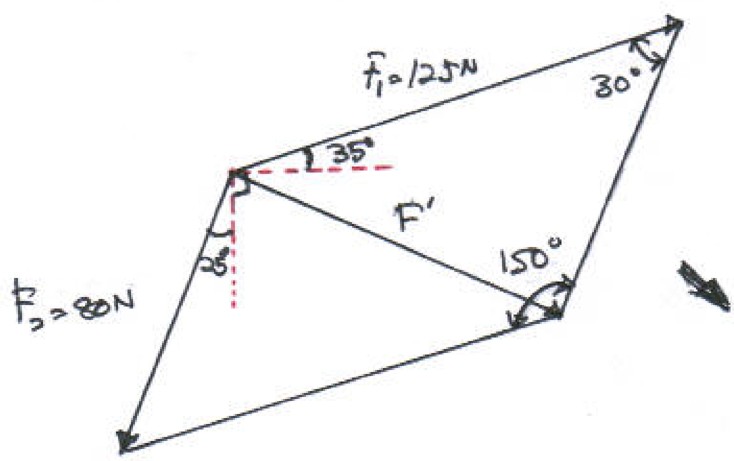
$$\underline{\underline{\theta = 25°}}$$

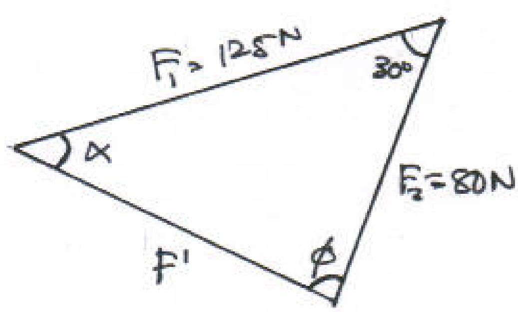
3) DETERMINE THE MAGNITUDE AND DIRECTION OF THE RESULTANT FORCE $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$.



- FIND THE RESULTANT $\vec{F}' = \vec{F}_1 + \vec{F}_2$ FIRST:

HENCE SKETCH THE PARALLELOGRAM FOR FORCE $\vec{F}_1$ & $\vec{F}_2$:





- USING COSINE LAW TO SOLVE F':

$$F' = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos 30^\circ}$$

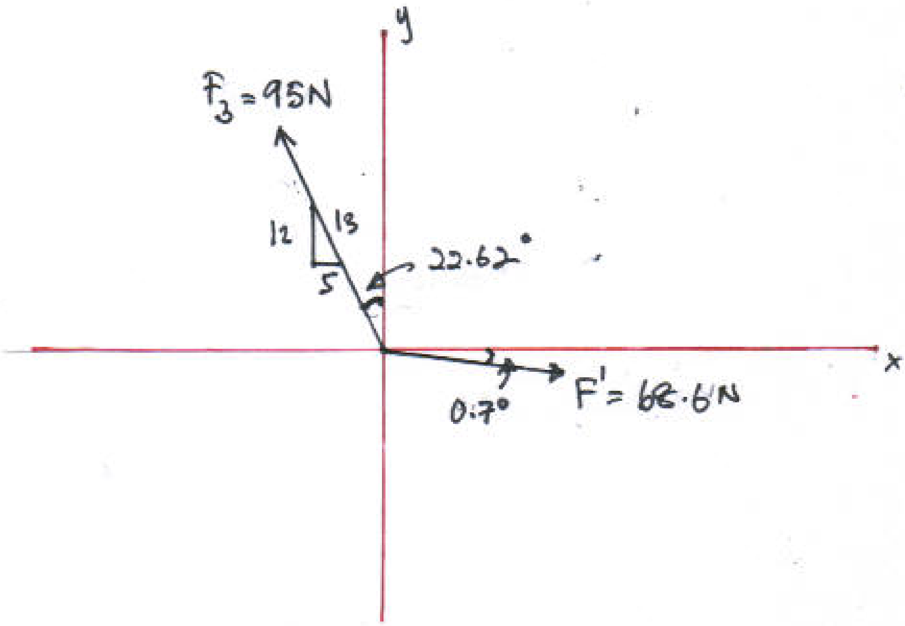
$$= \sqrt{125^2 + 80^2 - 2(125)(80)\cos 30^\circ}$$

$$F' = 68.6\text{ N}$$

USING SINE LAW TO SOLVE α:

$$\frac{80}{\sin \alpha} = \frac{68.6}{\sin 30^\circ}$$

$$\therefore \alpha = 35.7^\circ$$

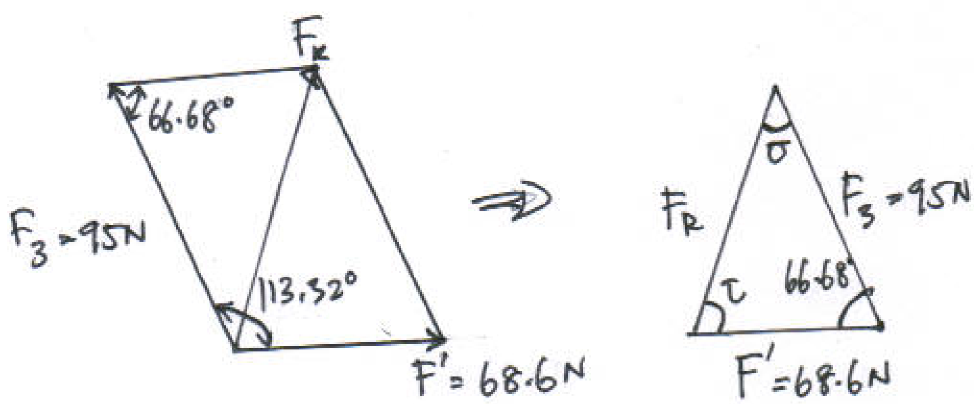


FIND THE RESULTANT

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_R = \vec{F}' + \vec{F}_3$$

SKETCH THE PARALLELOGRAM FOR FORCES F' & F_3



USING COSINE LAW TO SOLVE F_R:

$$F_R = \sqrt{(F')^2 + F_3^2 - 2(F')(F_3)\cos 66.68^\circ}$$

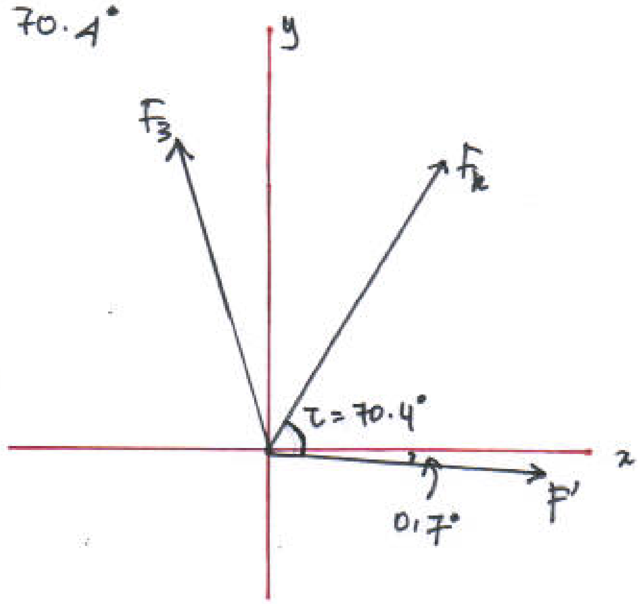
$$= \sqrt{68.6^2 + 95^2 - 2(68.6)(95)\cos 66.68}$$

$$\underline{\underline{F_R = 92.6\ N}}$$

USING SINE LAW TO SOLVE τ:

$$\frac{95}{\sin\tau} = \frac{92.6}{\sin 66.68}$$

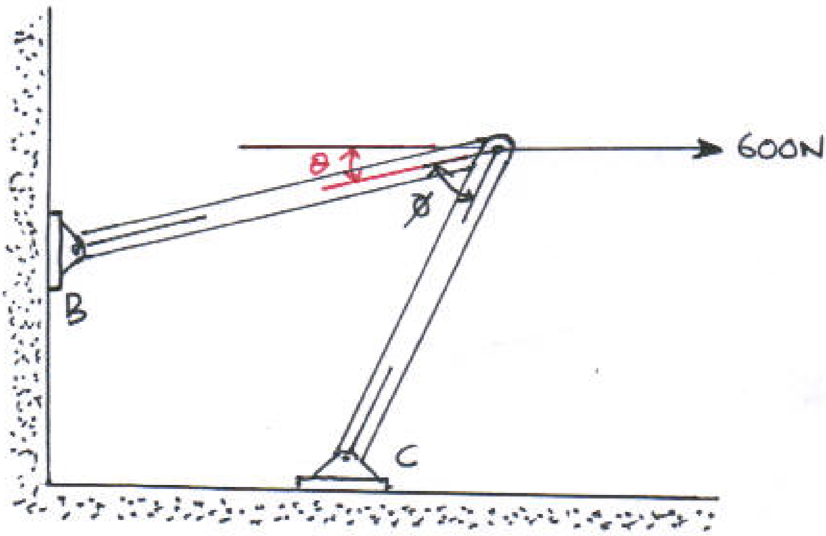
$$\tau = 70.4^\circ$$



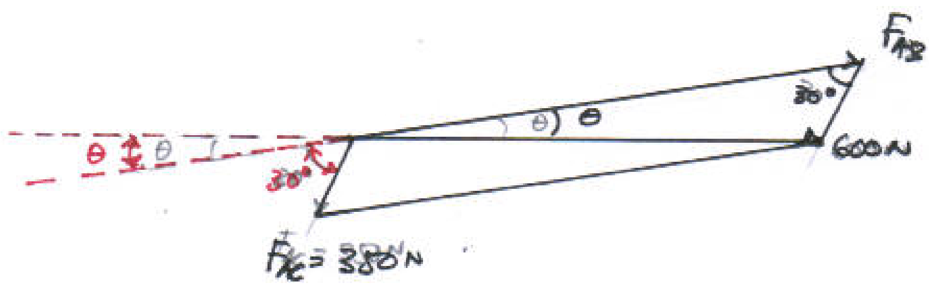
∴ DIRECTION OF F_R = $70.4^\circ - 0.7^\circ$

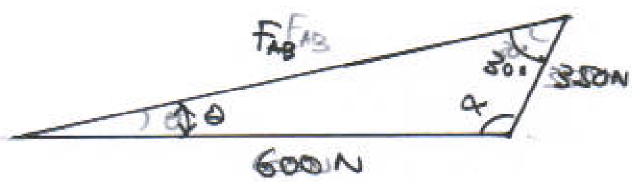
$= \underline{\underline{69.7^\circ \text{ ANTICLOCKWISE FROM POSITIVE X-AXIS.}}}$

A) DETERMINE THE DESIGN ANGLE θ $(0° \le \theta \le 90°)$ FOR STRUT AB SO THAT THE 600N HORIZONTAL FORCE HAS A COMPONENT OF 350N DIRECTED FROM A TOWARDS C. WHAT IS THE COMPONENT OF FORCE ACTING ALONG MEMBER AB? TAKE $\phi = 30°$



- SKETCH THE PARALLELOGRAM OF THE 600N FORCE !





- USE SINE LAW TO SOLVE θ

$$\frac{600}{\sin 30°} = \frac{350}{\sin\theta}$$

$$\therefore \theta = 17°$$

$\therefore \quad \alpha = 180^\circ - 30^\circ - 17^\circ$

$\alpha = 133^\circ$

USE COSINE LAW TO SOLVE F_{AB} :

$$F_{AB} = \sqrt{350^2 + 600^2 - 2(350)(600)\cos 133^\circ}$$

$$\underline{\underline{F_{AB} = 877\ N}}$$